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**ADVANCED GCE**

# MATHEMATICS

Further Pure Mathematics 3

**4727**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**
- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**
None

**Wednesday 20 May 2009**
**Afternoon**

**Duration:** 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

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3R–8H16

**1** Find the cube roots of $\frac{1}{2}\sqrt{3} + \frac{1}{2}\text{i}$, giving your answers in the form $\cos\theta + \text{i}\sin\theta$, where $0 \leqslant \theta < 2\pi$. **[4]**

**2** It is given that the set of complex numbers of the form $r\text{e}^{\text{i}\theta}$ for $-\pi < \theta \leqslant \pi$ and $r > 0$, under multiplication, forms a group.

**(i)** Write down the inverse of $5\text{e}^{\frac{1}{3}\pi\text{i}}$. **[1]**

**(ii)** Prove the closure property for the group. **[2]**

**(iii)** $Z$ denotes the element $\text{e}^{\text{i}\gamma}$, where $\frac{1}{2}\pi < \gamma < \pi$. Express $Z^2$ in the form $\text{e}^{\text{i}\theta}$, where $-\pi < \theta < 0$. **[2]**

**3** A line $l$ has equation $\dfrac{x-6}{-4} = \dfrac{y+7}{8} = \dfrac{z+10}{7}$ and a plane $p$ has equation $3x - 4y - 2z = 8$.

**(i)** Find the point of intersection of $l$ and $p$. **[3]**

**(ii)** Find the equation of the plane which contains $l$ and is perpendicular to $p$, giving your answer in the form $ax + by + cz = d$. **[5]**

**4** The differential equation

$$\frac{\text{d}y}{\text{d}x} + \frac{1}{1-x^2}y = (1-x)^{\frac{1}{2}}, \quad \text{where } |x| < 1,$$

can be solved by the integrating factor method.

**(i)** Use an appropriate result given in the List of Formulae (MF1) to show that the integrating factor can be written as $\left(\dfrac{1+x}{1-x}\right)^{\frac{1}{2}}$. **[2]**

**(ii)** Hence find the solution of the differential equation for which $y = 2$ when $x = 0$, giving your answer in the form $y = \text{f}(x)$. **[6]**

**5** The variables $x$ and $y$ satisfy the differential equation

$$\frac{\text{d}^2y}{\text{d}x^2} - 6\frac{\text{d}y}{\text{d}x} + 9y = \text{e}^{3x}.$$

**(i)** Find the complementary function. **[3]**

**(ii)** Explain briefly why there is no particular integral of either of the forms $y = k\text{e}^{3x}$ or $y = kx\text{e}^{3x}$. **[1]**

**(iii)** Given that there is a particular integral of the form $y = kx^2\text{e}^{3x}$, find the value of $k$. **[5]**

**6** The plane $\Pi_1$ has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix}$.

**(i)** Express the equation of $\Pi_1$ in the form $\mathbf{r.n} = p$. **[4]**

The plane $\Pi_2$ has equation $\mathbf{r.}\begin{pmatrix} 7 \\ 17 \\ -3 \end{pmatrix} = 21$.

**(ii)** Find an equation of the line of intersection of $\Pi_1$ and $\Pi_2$, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. **[5]**

**7** **(i)** Use de Moivre's theorem to prove that

$$\tan 3\theta \equiv \frac{\tan\theta(3 - \tan^2\theta)}{1 - 3\tan^2\theta}.$$ **[4]**

**(ii) (a)** By putting $\theta = \frac{1}{12}\pi$ in the identity in part **(i)**, show that $\tan\frac{1}{12}\pi$ is a solution of the equation

$$t^3 - 3t^2 - 3t + 1 = 0.$$ **[1]**

**(b)** Hence show that $\tan\frac{1}{12}\pi = 2 - \sqrt{3}$. **[4]**

**(iii)** Use the substitution $t = \tan\theta$ to show that

$$\int_0^{2-\sqrt{3}} \frac{t(3 - t^2)}{(1 - 3t^2)(1 + t^2)}\, dt = a\ln b,$$

where $a$ and $b$ are positive constants to be determined. **[5]**

**8** A multiplicative group $Q$ of order 8 has elements $\{e, p, p^2, p^3, a, ap, ap^2, ap^3\}$, where $e$ is the identity. The elements have the properties $p^4 = e$ and $a^2 = p^2 = (ap)^2$.

**(i)** Prove that $a = pap$ and that $p = apa$. **[2]**

**(ii)** Find the order of each of the elements $p^2, a, ap, ap^2$. **[5]**

**(iii)** Prove that $\{e, a, p^2, ap^2\}$ is a subgroup of $Q$. **[4]**

**(iv)** Determine whether $Q$ is a commutative group. **[4]**

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