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General Certificate of Education

Mathematics 6360

MS2B Statistics 2B

Mark Scheme

2006 examination – January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Question	Solution	Marks	Total	Comments
1(a)(i)	$\mathrm{P}(X=2)=\dfrac{\mathrm{e}^{-1.5}\times(1.5)^2}{2!}=0.251$	M1A1	2	
(ii)	$p=(0.251)^3=0.0158$	M1A1√	2	on their p from (i)
(b)(i)	$Y \sim \mathrm{P}_o(9.0)$	B1	1	
(ii)	$\mathrm{P}(Y\geq 12)=1-\mathrm{P}(Y\leq 11)$ $=1-0.8030$ $=0.197$	M1 A1	2	
(c)	attacks patients: randomly (p constant) independently	B1 B1	2	mean of 1.5 $\Rightarrow p$ small (B1) (unless very few patients)
	Total		**9**	
2(a)	H_o : Choice independent of gender	B1		gender not associated with choice
	Squash / Badminton / Archery / Hockey Male: 5/3.5, 16/14, 30/24.5, 19/28 Female: 4/5.5, 20/22, 33/38.5, 53/44	M1		
	Combine Squash and Badminton	M1		$\mathrm{E}_i<5$ (Similar categories)
	S & B / Archery / Hockey Male: 21/17.5, 30/24.5, 19/28 Female: 24/27.5, 33/38.5, 53/44	M1 M1		
	χ^2 values S & B / Archery / Hockey Male: 0.7000, 1.2347, 2.8928 Female: 0.4455, 0.7857, 1.8409	M1		
	$\chi^2_{\text{calc}}=7.90$	A1		(7.8 to 7.9)
	$\nu=2$	B1		
	$\chi^2_{5\%}(2)=5.991$	B1ft		(on their ν)
	Reject H_o Sufficient evidence, at the 1% level of significance, to support an association between the choice of sport and gender	A1ft	10	reject H_o and H_o stated **or** statement in context
(b)	**More females and fewer males** chose to participate in hockey **than expected**	B1 B1	2	
	Total		**12**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
3(a)	$\bar{x} = 8.0$	B1		
	$S = 2.121$	B1		
	$\nu = 8$	B1		
	$t = 1.860$	B1√		(on their ν)
	90% confidence interval for μ			
	$= 8 \pm 1.860\left(\frac{2.121}{3}\right)$	M1		
	$= 8 \pm 1.315$	A1ft		
	$= (6.68, 9.32)$	A1	7	(6.68 to 6.69, 9.31 to 9.32)
(b)	The Headteacher's claim seems to be slightly optimistic	E1ft		Headteacher's claim isn't supported by the evidence **and**
	because value of 5 outside the confidence interval	E1ft	2	It appears that the mean time to see a mathematics teacher is greater than 5 minutes
	Total		**9**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
4(a)(i)	Area $= k(b-a) = 1$			
	$\Rightarrow \quad k = \dfrac{1}{b-a}$	E1	1	AG
(ii)	$\mathrm{E}(X) = \int_a^b kx \, \mathrm{d}x$	M1		
	$= \left(\dfrac{kx^2}{2}\right)\Big\vert_a^b$	A1		
	$= \dfrac{1}{2}k\left(b^2 - a^2\right)$			
	$= \dfrac{1}{2} \times \dfrac{1}{(b-a)} \times (b-a)(a+b)$	M1A1		(factors shown)
	$= \dfrac{1}{2}(a+b)$		4	AG
(b)(i)	$\mu = 1$	B1	1	
(ii)	$\sigma^2 = \mathrm{Var}(X) = \dfrac{1}{12}(b-a)^2$			
	$= \dfrac{1}{12} \times 6^2$	M1		
	$= 3$			
	$\therefore \sigma = \sqrt{3}$	A1	2	1.7321
(iii)	$\mathrm{P}\left(X < \dfrac{2-\mu}{\sigma}\right) = \mathrm{P}\left(X < \dfrac{1}{\sqrt{3}}\right)$	M1$\surd$		(on their μ and σ)
	$= \dfrac{1}{6} \times 2.577$	M1$\surd$		
	$= 0.430$	A1	3	cao
	Total		**11**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
5(a)	$\mathrm{E}(X)=\sum_{\text{all }x} x\,\mathrm{P}(X=x)$			
	$=50$	B1		(cao)
	$\mathrm{E}(X^2)=\sum_{\text{all }x} x^2\mathrm{P}(X=x)$			
	$=2602.6(0)$	M1		
	$\mathrm{Var}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2$			
	$=2602.6-50^2$	M1		
	$=102.6(0)$			
	$\Rightarrow$ standard deviation (X) = 10.13	A1	4	(to nearest 1p)
(b)	$\mathrm{E}(Y)=\mu=\mathrm{E}(10X+250)$			
	$=10\times\mathrm{E}(X)+250$			
	$=750$	B1√		(on their $\mathrm{E}(X)$)
	$\mathrm{s.d}(Y)=10\times10.1$			
	$=101$	B1√	2	(on their $\mathrm{sd}(X)$)
	Total		**6**	
6(a)	$\mathrm{H_o}:\mu=65$			
	$\mathrm{H_1}:\mu<65$	B1		1-tailed test
	$\bar{X}\sim\mathrm{N}\left(65,\dfrac{81}{35}\right)$			
	$z_{crit}=-1.6449$	B1		
	$z=\dfrac{61.5-65}{9/\sqrt{35}}=-2.30$	M1A1		for σ^2/n used
	Reject $\mathrm{H_o}$ at 5% level of significance	A1√		(on their z-values)
	Evidence to suggest students may be under-achieving	E1	6	
(b)	Reject $\mathrm{H_o}$ when $\mathrm{H_o}$ true	E1		
	$\Downarrow$			
	Conclude that students are under-achieving when in fact they are not	E1	2	
	Total		**8**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
7(a)	$\mathrm{E}(T)=\int_0^1 t\ \mathrm{f}(t)\ \mathrm{d}t$			
	$=\int_0^1 4t^2\left(1-t^2\right)\mathrm{d}t$	M1		
	$=\left(\frac{4t^3}{3}-\frac{4t^5}{5}\right)\Big\vert_0^1$	A1		
	$=\frac{4}{3}-\frac{4}{5}$	A1		
	$=\frac{8}{15}$		3	AG
(b)(i)	$\mathrm{F}(t)=\mathrm{P}(T\le t)=\int_0^t \mathrm{f}(t)\,\mathrm{d}t$			
	$=\int_0^t 4t\left(1-t^2\right)\mathrm{d}t$	M1		
	$=\left(2t^2-t^4\right)\Big\vert_0^t$			
	$=2t^2-t^4$	A1	2	
(ii)	$\mathrm{P}(\mu<T<\mathrm{m})=\mathrm{F}(\mathrm{m})-\mathrm{F}(\mu)$	M1		
	$\Downarrow$			
	$\mathrm{F}(m)=0.5$	B1		
	$\mathrm{F}(\mu)=\mathrm{F}\left(\frac{8}{15}\right)=0.4880$	B1		
	$\therefore\ \mathrm{P}(\mu<T<\mathrm{m})=0.5-0.4880$	M1√		0.5 – their F(μ)
	$=0.012$	A1	5	
	Total		**10**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
8	$H_o : \mu = 1000$ $H_1 : \mu \neq 1000$	B1		2-tailed test
	$\bar{x} = \frac{12036}{12} = 1003$	B1		
	$S = 5.444$	B1		$(S^2 = 29.6)$
	$\nu = 12 - 1 = 11$	B1		
	$t = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{1003 - 1000}{5.444/\sqrt{12}} = 1.91$	M1 A1ft A1		
	$t_{crit} = \pm 2.201$	B1√		(on their ν)
	Accept H_o	A1√		(on their t-values)
	Insufficient evidence to indicate a change in the mean content of sherry in a bottle	E1√	10	
	Total		**10**	
	TOTAL		**75**	