General Certificate of Education
June 2006
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

Monday 22 May 2006 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

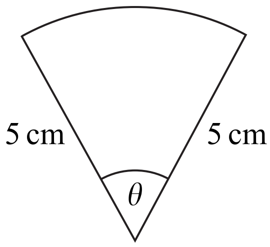
Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The diagram shows a sector of a circle of radius 5 cm and angle θ radians.

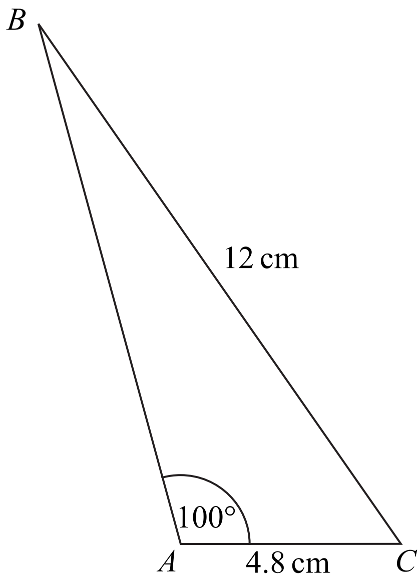


The area of the sector is $8.1\ \text{cm}^2$.

(a) Show that $\theta = 0.648$. *(2 marks)*

(b) Find the perimeter of the sector. *(3 marks)*

2 The diagram shows a triangle ABC.



The lengths of AC and BC are 4.8 cm and 12 cm respectively.

The size of the angle BAC is $100°$.

(a) Show that angle $ABC = 23.2°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

3 The first term of an arithmetic series is 1. The common difference of the series is 6.

(a) Find the tenth term of the series. *(2 marks)*

(b) The sum of the first n terms of the series is 7400.

(i) Show that $3n^2 - 2n - 7400 = 0$. *(3 marks)*

(ii) Find the value of n. *(2 marks)*

4 (a) The expression $(1 - 2x)^4$ can be written in the form

$$1 + px + qx^2 - 32x^3 + 16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers p and q. *(3 marks)*

(b) Find the coefficient of x in the expansion of $(2 + x)^9$. *(2 marks)*

(c) Find the coefficient of x in the expansion of $(1 - 2x)^4(2 + x)^9$. *(3 marks)*

5 (a) Given that

$$\log_a x = 2\log_a 6 - \log_a 3$$

show that $x = 12$. *(3 marks)*

(b) Given that

$$\log_a y + \log_a 5 = 7$$

express y in terms of a, giving your answer in a form not involving logarithms. *(3 marks)*

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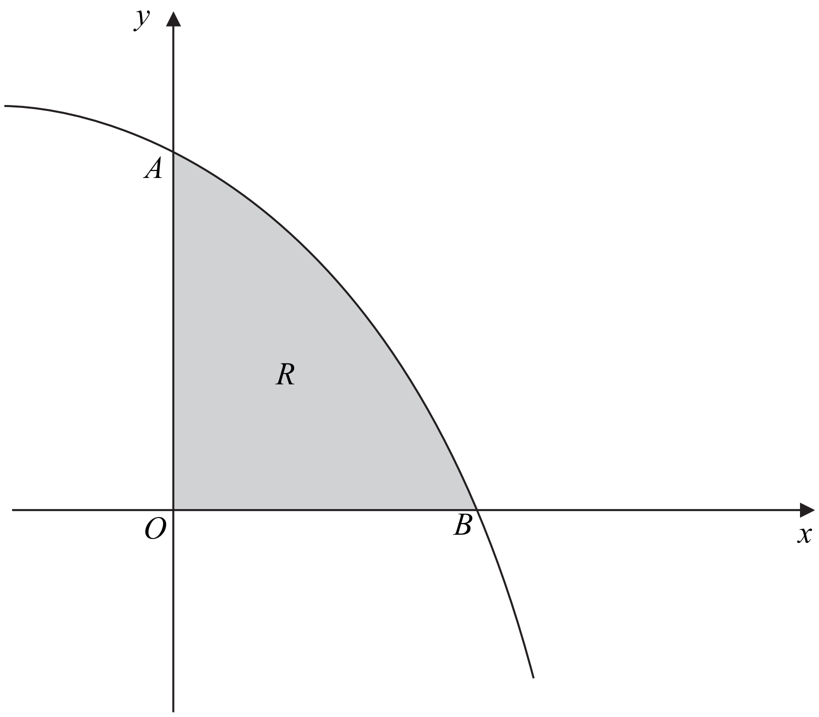
6 The diagram shows a sketch of the curve with equation $y = 27 - 3^x$.



The curve $y = 27 - 3^x$ intersects the y-axis at the point A and the x-axis at the point B.

(a) (i) Find the y-coordinate of point A. *(2 marks)*

(ii) Verify that the x-coordinate of point B is 3. *(1 mark)*

(b) The region, R, bounded by the curve $y = 27 - 3^x$ and the coordinate axes is shaded. Use the trapezium rule with four ordinates (three strips) to find an approximate value for the area of R. *(4 marks)*

(c) (i) Use logarithms to solve the equation $3^x = 13$, giving your answer to four decimal places. *(3 marks)*

(ii) The line $y = k$ intersects the curve $y = 27 - 3^x$ at the point where $3^x = 13$. Find the value of k. *(1 mark)*

(d) (i) Describe the single geometrical transformation by which the curve with equation $y = -3^x$ can be obtained **from** the curve $y = 27 - 3^x$. *(2 marks)*

(ii) Sketch the curve $y = -3^x$. *(2 marks)*

7 At the point (x, y), where $x > 0$, the gradient of a curve is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7$$

(a) (i) Verify that $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = 4$. *(1 mark)*

(ii) Write $\frac{16}{x^2}$ in the form $16x^k$, where k is an integer. *(1 mark)*

(iii) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. *(3 marks)*

(iv) Hence determine whether the point where $x = 4$ is a maximum or a minimum, giving a reason for your answer. *(2 marks)*

(b) The point $P(1, 8)$ lies on the curve.

(i) Show that the gradient of the curve at the point P is 12. *(1 mark)*

(ii) Find an equation of the normal to the curve at P. *(3 marks)*

(c) (i) Find $\int \left(3x^{\frac{1}{2}} + \frac{16}{x^2} - 7\right) \mathrm{d}x$. *(3 marks)*

(ii) Hence find the equation of the curve which passes through the point $P(1, 8)$. *(3 marks)*

8 (a) Describe the single geometrical transformation by which the curve with equation $y = \tan\frac{1}{2}x$ can be obtained from the curve $y = \tan x$. *(2 marks)*

(b) Solve the equation $\tan\frac{1}{2}x = 3$ in the interval $0 < x < 4\pi$, giving your answers in radians to three significant figures. *(4 marks)*

(c) Solve the equation

$$\cos\theta(\sin\theta - 3\cos\theta) = 0$$

in the interval $0 < \theta < 2\pi$, giving your answers in radians to three significant figures. *(5 marks)*

END OF QUESTIONS

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