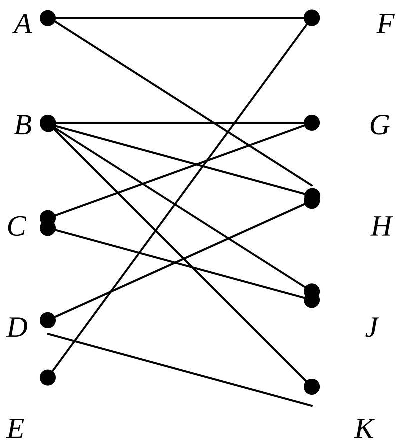
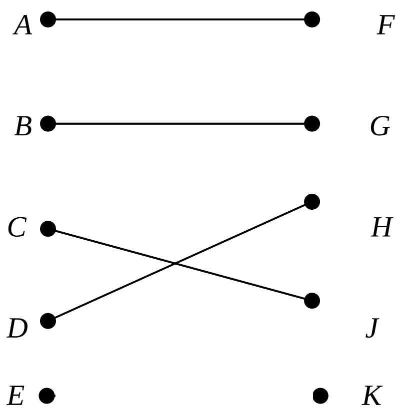
4737 Decision Mathematics 2

1(a) (i)		B1	A correct bipartite graph	[1]
(ii)		B1	A second bipartite graph showing the incomplete matching correctly	[1]
(iii)	$E = F – A = H – D = K$ Fiona = Egg and tomato $F = E$ Gwen = Beef and horseradish $G = B$ Helen = Avocado and bacon $H = A$ Jack = Chicken and stuffing $J = C$ Mr King = Duck and plum sauce $K = D$	B1 B1	This path in any reasonable form This complete matching	[2]
(iv)	Interchange Gwen and Jack $F = E$ $G = C$ $H = A$ $J= B$ $K = D$	B1	This complete matching	[1]

(b)

Reduce rows

	F	*G*	*H*	*J*	*K*
L	7	7	7	7	0
M	2	6	4	2	0
N	8	8	8	6	0
O	1	3	2	1	0
P	6	9	7	5	0

M1 Substantially correct attempt to reduce rows

Reduce columns

	F	*G*	*H*	*J*	*K*
L	6	4	5	6	0
M	1	3	2	1	0
N	7	5	6	5	0
O	0	0	0	0	0
P	5	6	5	4	0

M1 Substantially correct attempt to reduce columns

A1 cao

[3]

Cross out 0's using two (minimum no. of) lines

	F	*G*	*H*	*J*	*K*
L	6	4	5	6	**0**
M	1	3	2	1	**0**
N	7	5	6	5	**0**
O	**0**	**0**	**0**	**0**	**0**
P	5	6	5	4	**0**

Augment by 1

M1 Substantially correct attempt at augmenting

	F	*G*	*H*	*J*	*K*
L	5	3	4	5	0
M	0	2	1	0	0
N	6	4	5	4	0
O	0	0	0	0	1
P	4	5	4	3	0

A1 Augmenting correctly

[2]

Cross out 0's using three (minimum no. of) lines

	F	*G*	*H*	*J*	*K*
L	5	3	4	5	**0**
M	**0**	**2**	**1**	**0**	**0**
N	6	4	5	4	**0**
O	**0**	**0**	**0**	**0**	**1**
P	4	5	4	3	**0**

Augment by 3

	F	*G*	*H*	*J*	*K*
L	2	**0**	1	2	0
M	**0**	2	1	0	3
N	3	1	2	1	**0**
O	0	0	**0**	0	4
P	1	2	1	**0**	0

M1 Substantially correct attempt at augmenting (by more than 1 in a single step)

A1 Augmenting correctly

Lemon = Gwen
Mandarin = Fiona
Nectarine = Mr King
Orange = Helen
Peach = Jack

B1 Correct allocation

[3]

Total = 13

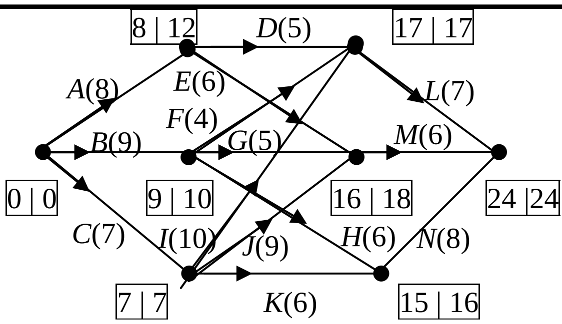
2 (i)

Stage	State	Action	Working	Suboptimal maxima
2	0	0	7	7
	1	0	6	6
	2	0	8	8
1	0	0	5 +7 = 12	
		1	6 + 6 = 12	12
	1	0	4 +7 = 11	
		1	5 + 6 = 11	
		2	6 + 8 = 14	14
	2	0	10 + 7 = 17	17
		1	9 + 6 = 15	
		2	6 + 8 = 14	
0	0	0	8 + 12 = 20	
		1	9 + 14 = 23	
		2	7 + 17 = 24	24

Maximum route = (0;0) - (1;2) - (2;0) - (3;0)
Weight = 24

B1	Structure of table correct	
M1	Stage and state values correct	
A1	Action values correct	[3]
B1	Working backwards from stage 2 7, 6, 8 correct in suboptimal maxima column for stage 2	
M1	Working column substantially correct for stage 1	
A1	Sums correct for stage 1	[3]
B1	Suboptima maxima values correct for stage 1	
M1	Working column substantially correct for stage 0	
A1	Sums correct for stage 0	
		[3]
B1	Correct route from (0; 0) to (3; 0)	
B1	24 cao	[2]

(ii)



Minimum completion time = 24
Critical activities: *C*, *I*, *L*

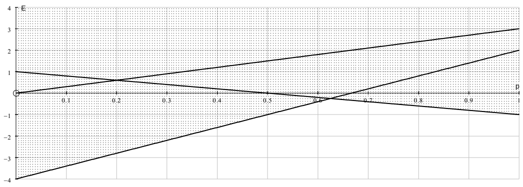
B1	Assigning *A* to *N* appropriately	
M1	Substantially correct forward pass	
A1	Forward pass correct	
M1	Substantially correct backward pass	
	Backward pass correct	
A1	24 (cao)	
B1	*C*, *I*, *L* (cao)	
B1		[7]

(iii)

The critical path is the maximum path
The critical activities form a continuous path with no slack, ie the longest path

M1	Same path is found in both	
A1	Recognition of why the solutions are the same, in general	[2]

Total = **20**

3 (i)	For each pairing, the total of the points is 10. Subtracting 5 from each makes the total 0. Eg 3 points and 7 points ⇒ scores of -2 and +2	M1 A1	Sum of points is 10 So sum of scores is zero A specific example earns M1 only	[2]
(ii)	*W* scores -1 *P* has 6 points and *W* has 4 points	B1 B1	-1 6 and 4	[2]
(iii)	*W* is dominated by *Y* -1 < 1, -3 < -2 and 1 < 2	B1 B1	*Y* These three comparisons in any form	[2]
(iv)	Collies $\begin{array}{ll\|c\|c\|cl} & & X & Y & Z & \text{row min} \\ \text{Rovers} & P & 2 & -1 & 3 & -1 \\ & Q & 1 & -3 & -1 & -3 \\ & R & -4 & 1 & 0 & -4 \\ \text{col max} & & 2 & 1 & 3 & \end{array}$ Play-safe for Rovers is *P* Play-safes for Collies is *Y*	M1 A1 A1	Determining row minima and column maxima, or equivalent *P* *Y*	[3]
(v)	$2p - 4(1-p) = 6p - 4$ *Y* gives $1 - 2p$ *Z* gives $3p$	B1 B1	$6p - 4$ in simplified form Both $1 - 2p$ and $3p$ in any form	[2]
(vi)	 $6p - 4 = 1 - 2p \Rightarrow p = \frac{5}{8}$	B1 M1 A1	Their lines drawn correctly on a reasonable scale Solving the correct pair of equations or using graph correctly $\frac{5}{8}$, 0.625, cao	[3]
(vii)	Add 4 throughout matrix to make all values non-negative On this augmented matrix, if Collies play *X* Rovers expect $6p_1 + 5p_2$; if Collies play *Y* Rovers expect $3p_1 + p_2 + 5p_3$; and if Collies play *Z* Rovers expect $7p_1 + 3p_2 + 4p_3$ We want to maximise *M* where *M* only differs by a constant from *m* and, for each value of *p*, *m* is the minimum expected value.	B1 B1 B1	'Add 4', or new matrix written out or equivalent Relating to columns *X*, *Y* and *Z* respectively. Note: expressions are given in the question. For each value of *p* we look at the minimum output, then we maximise these minima.	[3]
(viii)	$p_3 = \frac{3}{8}$ $M = -\frac{1}{4}$	B1 B1	cao cao	[2]
			Total =	**19**

4 (i)	8+0+6+5+4 = 23 gallons per minute	M1 A1	8+0+6+5+4 or 23 23 with units	[2]
(ii)	At most 6 gallons per minute can enter *A* so there cannot be 7 gallons per minute leaving it At most 7 gallons per minute can leave *F* so there cannot be 10 gallons per minute entering it.	B1 B1	Maximum into $A = 6$ Maximum out of $F = 7$	[2]
(iii)	A diagram showing a flow with 12 through *E* Flow is feasible (upper capacities not exceeded) Nothing flows through *A* and *D* Maximum flow through $E = 12$ gallons per minute	M1 M1 A1 B1	Assume that blanks mean 0 12	[4]
(iv) a **b**	If flows through *A* but not *D* its route must be $S – A – C – E$, but the flow through *E* is already a maximum $S – (B) – C – D – F – T$ 1 gallon per minute	B1 M1 A1	A correct explanation Follow through their part (iii) 1	[1] [2]
(v)	Flow = 12 + 1 = 13 gallons per minute Cut through *ET* and *FT* or $\{S,A,B,C,D,E,F\}$, $\{T\}$ = 13 gallons per minute Every cut forms a restriction Every cut $\geq$ every flow min cut $\geq$ max flow This cut = this flow so must be min cut and max flow	B1 M1 A1 B1	Identifying this cut in any way Use of max flow – min cut theorem min cut $\geq$ max flow This cut = this flow (or having shown that both are 13)	[4]
(vi)	3 gallons per minute Must flow 6 along *ET* and 7 along *FT*. Can send 4 into *F* from *D* so only need to send 9 through *E*	B1 B1 B1	3 A correct explanation	[3]
(vii)	A diagram showing a flow of 13 without using *BE* Flow is feasible and only sends 9 through *E*	M1 A1	May imply directions and assume that blanks mean 0	[2]
			Total =	**20**