General Certificate of Education
Advanced Subsidiary Examination
January 2013

Mathematics MPC2

Unit Pure Core 2

Monday 14 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

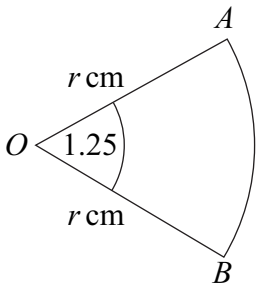
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The diagram shows a sector OAB of a circle with centre O and radius r cm.



The angle AOB is 1.25 radians. The perimeter of the sector is 39 cm.

(a) Show that $r = 12$. *(3 marks)*

(b) Calculate the area of the sector OAB. *(2 marks)*

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2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

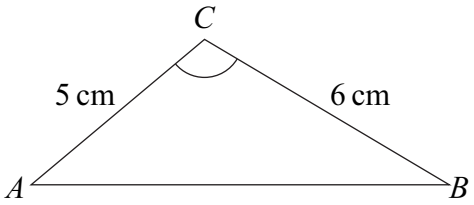
$$\int_1^5 \frac{1}{x^2+1}\,\mathrm{d}x$$

giving your answer to three significant figures. *(4 marks)*

(b) (i) Find $\int\left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}}\right)\mathrm{d}x$, giving the coefficient of each term in its simplest form. *(3 marks)*

(ii) Hence find the value of $\int_1^4\left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}}\right)\mathrm{d}x$. *(2 marks)*

3 The diagram shows a triangle ABC.



The lengths of AC and BC are 5 cm and 6 cm respectively.

The area of triangle ABC is $12.5\,\text{cm}^2$, and angle ACB is **obtuse**.

(a) Find the size of angle ACB, giving your answer to the nearest 0.1°. *(3 marks)*

(b) Find the length of AB, giving your answer to two significant figures. *(3 marks)*

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4 Given that

$$\log_a N - \log_a x = \frac{3}{2}$$

express x in terms of a and N, giving your answer in a form not involving logarithms. *(3 marks)*

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5 The point $P(2, 8)$ lies on a curve, and the point M is the only stationary point of the curve.

The curve has equation $y = 6 + 2x - \frac{8}{x^2}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(b) Show that the normal to the curve at the point $P(2, 8)$ has equation $x + 4y = 34$. *(3 marks)*

(c) (i) Show that the stationary point M lies on the x-axis. *(3 marks)*

(ii) Hence **write down** the equation of the tangent to the curve at M. *(1 mark)*

(d) The tangent to the curve at M and the normal to the curve at P intersect at the point T. Find the coordinates of T. *(2 marks)*

Turn over ▶

6 (a) A geometric series begins $420 + 294 + 205.8 + ...$

(i) Show that the common ratio of the series is 0.7. *(1 mark)*

(ii) Find the sum to infinity of the series. *(2 marks)*

(iii) Write the nth term of the series in the form $p \times q^n$, where p and q are constants. *(2 marks)*

(b) The first term of an arithmetic series is 240 and the common difference of the series is -8.

The nth term of the series is u_n.

(i) Write down an expression for u_n. *(1 mark)*

(ii) Given that $u_k = 0$, find the value of $\sum_{n=1}^{k} u_n$. *(4 marks)*

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7 (a) Describe a geometrical transformation that maps the graph of $y = 4^x$ onto the graph of $y = 3 \times 4^x$. *(2 marks)*

(b) Sketch the curve with equation $y = 3 \times 4^x$, indicating the value of the intercept on the y-axis. *(2 marks)*

(c) The curve with equation $y = 4^{-x}$ intersects the curve $y = 3 \times 4^x$ at the point P. Use logarithms to find the x-coordinate of P, giving your answer to three significant figures. *(5 marks)*

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8 (a) Expand $\left(1 + \frac{4}{x}\right)^2$. *(1 mark)*

(b) The first four terms of the binomial expansion of $\left(1 + \frac{x}{4}\right)^8$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the constants a, b and c. *(4 marks)*

(c) Hence find the coefficient of x in the expansion of $\left(1 + \frac{4}{x}\right)^2\left(1 + \frac{x}{4}\right)^8$. *(4 marks)*

9 (a) Write down the two solutions of the equation $\tan(x + 30°) = \tan 79°$ in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(b) Describe a single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x + 30°)$. *(2 marks)*

(c) (i) Given that $5 + \sin^2\theta = (5 + 3\cos\theta)\cos\theta$, show that $\cos\theta = \frac{3}{4}$. *(5 marks)*

(ii) **Hence** solve the equation $5 + \sin^2 2x = (5 + 3\cos 2x)\cos 2x$ in the interval $0 < x < 2\pi$, giving your values of x in radians to three significant figures. *(3 marks)*