4730 Mechanics 3

1	(i) $T = (1.35mg)(3 - 1.8) \div 1.8$	B1		
	$[0.9mg = ma]$	M1		For using $T = ma$
	Acceleration is $8.82ms^{-2}$	A1	3	
	(ii) Initial EE = $(1.35mg)(3 - 1.8)^2 \div (2x1.8)$	B1		
	$[\frac{1}{2} mv^2 = 0.54mg]$	M1		For using $\frac{1}{2} mv^2$ = Initial EE
	Speed is $3.25ms^{-1}$	A1	3	

2	**(i)**	M1		For using NEL vertically
	Component is $8esin27^o$	A1		
	Component is $2.18ms^{-1}$	A1	3	
	(ii) Change in velocity vertically = $8sin27^o(1 + e)$	B1ft		ft $8sin27^o$ + candidate's ans. in (i)
	$\|I\| = 0.2 \times 5.81$	M1		For using $\|I\|$ = m x change in velocity
	Magnitude of Impulse is 1.16 $kgms^{-1}$	A1ft	3	ft incorrect ans. in (i) providing both M marks are scored.

3		M1		For using the principle of conservation of momentum in the **i** direction
	$0.8x12cos60^o = 0.8a + 2b$	A1		
		M1		For using NEL
	$0.75x12cos60^o = b - a$	A1		
	$[4.8 = 0.8a + 2(a + 4.5)]$	DM1		For eliminating b; depends on at least one previous M mark
	$a = -1.5$	A1		
	Comp. of vel. perp. to l.o.c. after impact is $12sin60^o$	B1		
		M1		For correct method for speed or direction
	The speed of A is $10.5ms^{-1}$	A1ft		ft $v^2 = a^2 + 108$
	Direction of A is at 98.2^o to l.o.c.	A1ft	10	Accept $\theta = 81.8^o$ if θ is clearly and appropriately indicated; ft $\tan^{-1}\theta = (12sin60^o)/\|a\|$)

4	**(i)** $[mg\sin\alpha - 0.2mv = ma]$	M1		For using Newton's second law
	$5\frac{dv}{dt} = 28 - v$	A1		AG
	$[\int \frac{5}{28-v}dv = \int dt]$	M1		For separating variables and integrating
	$(C) - 5\ln(28 - v) = t$	A1		
		M1		For using v = 0 when t = 0
	$\ln[(28 - v)/28] = -t/5$	A1ft		ft for $\ln[(28 - v)/28] = t/A$ from $C + A\ln(28 - v) = t$ previously
	$[28 - v = 28e^{-t/5}]$	M1		For expressing v in terms of t
	$v = 28(1 - e^{-t/5})$	A1ft	8	ft for $v = 28(1 - e^{t/A})$ from $\ln[(28 - v)/28] = t/A$ previously
	(ii)			For using $a = (28 - v(t))/5$ or $a = d(28 - 28e^{-t/5})dt$ and substituting t = 10.
	$[a = 28e^{-2}/5]$	M1		ft from incorrect v in the form $a + be^{ct}$ $(b \neq 0)$; Accept $5.6/e^2$
	Acceleration is $0.758 ms^{-2}$	A1ft	2	

5	**(i)**			For taking moments about B or about A for the whole or For taking moments about X for the whole and using $R_A + R_B = 280$ and $F_A = F_B$
		M1		
	$1.4R_A = 150x0.95 + 130x0.25$ or $1.4R_B = 130x1.15 + 150x0.45$ or $1.2F - 0.9(280 - R_B) + 0.45x150 - 1.2F + 0.5R_B - 0.25x130 = 0$	A1		
	$R_A = 125N$	A1		AG
	$R_B = 155N$	B1	4	
	(ii)			For taking moments about X for XA or XB
		M1		
	$1.2F_A = -150x0.45 + 0.9R_A$ or $1.2F_B = 0.5R_B - 130x0.25$	A1		
	F_A or $F_B = 37.5N$	A1ft		$F_B = (1.25R_B - 81.25)/3$
	F_B or $F_A = 37.5N$	B1ft	4	
	(iii) Horizontal component is 37.5N to the left	B1ft		ft H = F or $H = 56.25 - 0.75V$ or $12H = 325 + 5V$
	$[Y + R_A = 150]$	M1		For resolving forces on XA vertically
	Vertical component is 25N upwards	A1ft	3	ft $3V = 225 - 4H$ or $V = 2.4H - 65$

6	**(i)**			For applying Newton's second law
	[0.36 – 0.144x = 0.1a]	M1		
	$\ddot{x} = 3.6 - 1.44x$	A1		
	$\ddot{y} = -1.44y$ ➔ SHM or $d^2(x-2.5)/dt^2 = -1.44(x-2.5)$ ➔SHM	B1		
		M1		For using $T = 2\pi/n$
	Of period 5.24s	A1	5	AG
	(ii) Amplitude is 0.5m	B1		
		M1		For using $v^2 = n^2(a^2 - y^2)$
	$0.48^2 = 1.2^2(0.5^2 - y^2)$	A1ft		
	Possible values are 2.2 and 2.8	A1	4	
	(iii) [$t_0 = (\sin^{-1}0.6)/1.2$; $t_1 = (\cos^{-1}0.6)/1.2$]	M1		For using $y = 0.5\sin 1.2t$ to find t_0 or $y = 0.5\cos 1.2t$ to find t_1
	$t_0 = 0.53625$ .... or $t_1 = 0.7727$…..	A1		Principal value may be implied
	(a)			For using $\Delta t = 2t_0$ or $\Delta t = T/2 - 2t_1$
	[$2(\sin^{-1}0.6)/1.2$ or $(\pi - 2\cos^{-1}0.6)/1.2$]	M1		
	Time interval is 1.07s	A1ft		ft incorrect t_0 or t_1
	(b)			From $\Delta t = T/2 - 2t_0$ or $\Delta t = 2t_1$; ft 2.62 – ans(a) or incorrect t_0 or t_1
	Time interval is 1.55s	B1ft	5	

7	**(i)**	M1		For using KE gain = PE loss
	½ $mv^2 = mga(1 - \cos\theta)$	A1		
	$aw^2 = 2g(1 - \cos\theta)$	B1	3	AG From v = wr
	(ii)			For using Newton's second law radially (3 terms required) with accel = v^2/r or w^2r
	$mv^2/a = mg\cos\theta - R$ or $maw^2 = mg\cos\theta - R$	M1		
		A1		
	[$2mg(1 - \cos\theta) = mg\cos\theta - R$]	DM1		For eliminating v^2 or w^2; depends on at least one previous M1
	$R = mg(3\cos\theta - 2)$	A1ft	4	ft sign error in N2 equation
	(iii)			
	[$mg\sin\theta$ = m(accel.) or $2a(\dot{\theta})\ddot{\theta} = 2g\sin\theta(\dot{\theta})$]	M1		For using Newton's second law tangentially or differentiating $aw^2 = 2g(1 - \cos\theta)$ w.r.t. t
	Accel. (=$a\ddot{\theta}$) = $g\sin\theta$	A1		
	[$\theta = \cos^{-1}(2/3)$]	M1		For using R = 0
	Acceleration is $7.30 ms^{-2}$	A1ft	4	ft from incorrect R of the form $mg(A\cos\theta + B)$, $A \neq 0$, $B \neq 0$; accept $g\sqrt{5}/3$
	(iv)			
		M1		For using rate of change = $(dR/d\theta)(d\theta/dt)$
	$dR/dt = (-3mg\sin\theta)\sqrt{2g(1-\cos\theta)/a}$	A1ft		ft from incorrect R of the form $mg(A\cos\theta + B)$, $A \neq 0$
		M1		For using $\cos\theta = 2/3$
	Rate of change is $-mg\sqrt{\frac{10g}{3a}}$ Ns^{-1}	A1ft	4	Any correct form of $\dot{R}$ with $\cos\theta = 2/3$ used; ft with θ from incorrect R of the form $mg(A\cos\theta + B)$, $A \neq 0$, $B \neq 0$