



ADVANCED GCE
MATHEMATICS

4730/01

Mechanics 3

THURSDAY 17 JANUARY 2008

Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of 4 printed pages.

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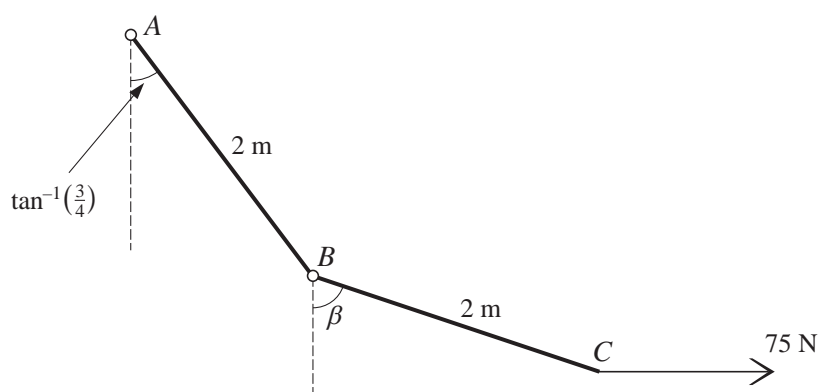
- 1 A smooth horizontal surface lies in the x - y plane. A particle P of mass 0.5 kg is moving on the surface with speed 5 m s^{-1} in the x -direction when it is struck by a horizontal blow whose impulse has components -3.5 N s and 2.4 N s in the x -direction and y -direction respectively.

- (i) Find the components in the x -direction and the y -direction of the velocity of P immediately after the blow. Hence show that the speed of P immediately after the blow is 5.2 m s^{-1} . [4]

P is struck by a second horizontal blow whose impulse is \mathbf{I} .

- (ii) Given that P 's direction of motion immediately after this blow is parallel to the x -axis, write down the component of \mathbf{I} in the y -direction. [2]

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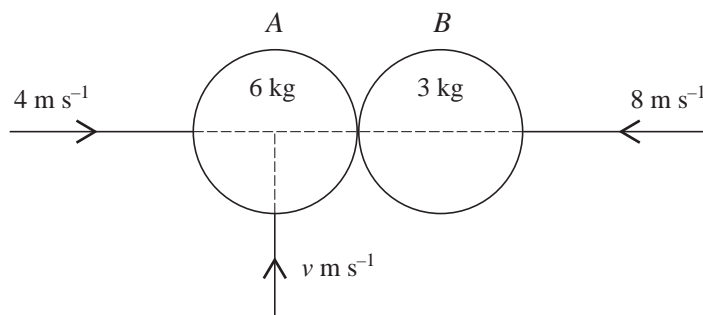


Two uniform rods AB and BC , each of length 2 m , are freely jointed at B . The weights of the rods are $W \text{ N}$ and 50 N respectively. The end A of AB is hinged at a fixed point. The rods AB and BC make angles $\tan^{-1}(\frac{3}{4})$ and β respectively with the downward vertical, and are held in equilibrium in a vertical plane by a horizontal force of magnitude 75 N acting at C (see diagram).

- (i) By taking moments about B for BC , show that $\tan \beta = 3$. [3]
- (ii) Write down the horizontal and vertical components of the force acting on AB at B . [2]
- (iii) Find the value of W . [4]

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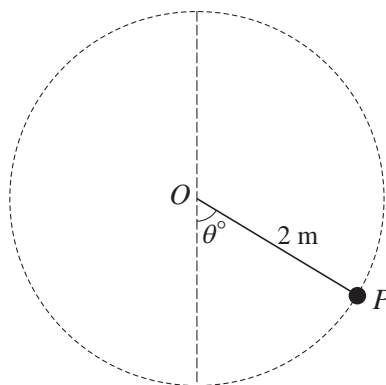
Two uniform smooth spheres A and B , of equal radius, have masses 6 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision the velocity of A has components 4 m s^{-1} along the line of centres towards B , and $v\text{ m s}^{-1}$ perpendicular to the line of centres. B is moving with speed 8 m s^{-1} along the line of centres towards A (see diagram). The coefficient of restitution between the spheres is e .

- (i) Find, in terms of e , the component of the velocity of A along the line of centres immediately after the collision. [5]
- (ii) Given that the speeds of A and B are the same immediately after the collision, and that $3e^2 = 1$, find v . [4]
- 4 A particle of mass $m\text{ kg}$ is released from rest at a fixed point O and falls vertically. The particle is subject to an upward resisting force of magnitude $0.49mv\text{ N}$ where $v\text{ m s}^{-1}$ is the velocity of the particle when it has fallen a distance of $x\text{ m}$ from O .
- (i) Write down a differential equation for the motion of the particle, and show that the equation can be written as $\left(\frac{20}{20-v} - 1\right)\frac{dv}{dx} = 0.49$. [5]
- (ii) Hence find an expression for x in terms of v . [5]
- 5 A particle P of mass $m\text{ kg}$ is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity $0.75mg\text{ N}$. The other end of the string is attached to a fixed point O of a smooth plane inclined at 30° to the horizontal. P is released from rest at O and moves down the plane.
- (i) Show that the maximum speed of P is reached when the extension of the string is 0.8 m . [3]
- (ii) Find the maximum speed of P . [4]
- (iii) Find the maximum displacement of P from O . [4]

[Questions 6 and 7 are printed overleaf.]

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6



A particle P of mass 0.4 kg is attached to one end of a light inextensible string of length 2 m . The other end of the string is attached to a fixed point O . With the string taut the particle is travelling in a circular path in a vertical plane. The angle between the string and the downward vertical is θ° (see diagram). When $\theta = 0$ the speed of P is 7 m s^{-1} .

(i) At the instant when the string is horizontal, find the speed of P and the tension in the string. [4]

(ii) At the instant when the string becomes slack, find the value of θ . [8]

7 A particle P , of mass $m \text{ kg}$, is attached to one end of a light elastic string of natural length 3.2 m and modulus of elasticity $4mg \text{ N}$. The other end of the string is attached to a fixed point A . The particle is released from rest at a point 4.8 m vertically below A . At time $t \text{ s}$ after P 's release P is $(4 + x) \text{ m}$ below A .

(i) Show that $4 \frac{d^2x}{dt^2} = -49x$. [3]

P 's motion is simple harmonic.

(ii) Write down the amplitude of P 's motion and show that the string becomes slack instantaneously at intervals of approximately 1.8 s . [4]

A particle Q is attached to one end of a light **inextensible** string of length $L \text{ m}$. The other end of the string is attached to a fixed point B . The particle is released from rest with the string taut and inclined at a small angle with the downward vertical. At time $t \text{ s}$ after Q 's release BQ makes an angle of θ radians with the downward vertical.

(iii) Show that $\frac{d^2\theta}{dt^2} \approx -\frac{g}{L}\theta$. [3]

The period of the simple harmonic motion to which Q 's motion approximates is the same as the period of P 's motion.

(iv) Given that $\theta = 0.08$ when $t = 0$, find the speed of Q when $t = 0.25$. [5]

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