General Certificate of Education
Advanced Subsidiary Examination
January 2011

Mathematics MPC1

Unit Pure Core 1

Monday 10 January 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The curve with equation $y = 13 + 18x + 3x^2 - 4x^3$ passes through the point P where $x = -1$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Show that the point P is a stationary point of the curve and find the other value of x where the curve has a stationary point. *(3 marks)*

(c) (i) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P. *(3 marks)*

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. *(1 mark)*

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2 (a) Simplify $\left(3\sqrt{3}\right)^2$. *(1 mark)*

(b) Express $\dfrac{4\sqrt{3} + 3\sqrt{7}}{3\sqrt{3} + \sqrt{7}}$ in the form $\dfrac{m + \sqrt{21}}{n}$, where m and n are integers. *(4 marks)*

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3 The line AB has equation $3x + 2y = 7$. The point C has coordinates $(2, -7)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) The line which passes through C and which is parallel to AB crosses the y-axis at the point D. Find the y-coordinate of D. *(3 marks)*

(b) The line with equation $y = 1 - 4x$ intersects the line AB at the point A. Find the coordinates of A. *(3 marks)*

(c) The point E has coordinates $(5, k)$. Given that CE has length 5, find the two possible values of the constant k. *(3 marks)*

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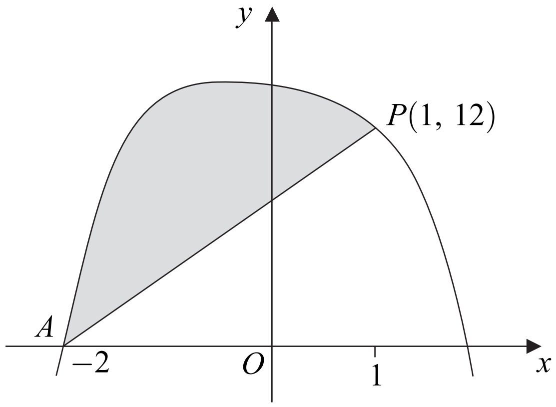
4 The curve sketched below passes through the point $A(-2, 0)$.



The curve has equation $y = 14 - x - x^4$ and the point $P(1, 12)$ lies on the curve.

(a) (i) Find the gradient of the curve at the point P. *(3 marks)*

(ii) Hence find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(2 marks)*

(b) (i) Find $\int_{-2}^{1} (14 - x - x^4)\, dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve $y = 14 - x - x^4$ and the line AP. *(2 marks)*

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5 (a) (i) Sketch the curve with equation $y = x(x-2)^2$. *(3 marks)*

(ii) Show that the equation $x(x-2)^2 = 3$ can be expressed as

$$x^3 - 4x^2 + 4x - 3 = 0$$ *(1 mark)*

(b) The polynomial $p(x)$ is given by $p(x) = x^3 - 4x^2 + 4x - 3$.

(i) Find the remainder when $p(x)$ is divided by $x + 1$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(iii) Express $p(x)$ in the form $(x-3)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(c) Hence show that the equation $x(x-2)^2 = 3$ has only one real root and state the value of this root. *(3 marks)*

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6 A circle has centre $C(-3,\ 1)$ and radius $\sqrt{13}$.

(a) (i) Express the equation of the circle in the form

$$(x-a)^2+(y-b)^2=k$$ *(2 marks)*

(ii) Hence find the equation of the circle in the form

$$x^2+y^2+mx+ny+p=0$$

where m, n and p are integers. *(3 marks)*

(b) The circle cuts the y-axis at the points A and B. Find the distance AB. *(3 marks)*

(c) (i) Verify that the point $D(-5,\ -2)$ lies on the circle. *(1 mark)*

(ii) Find the gradient of CD. *(2 marks)*

(iii) Hence find an equation of the tangent to the circle at the point D. *(2 marks)*

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7 (a) (i) Express $4-10x-x^2$ in the form $p-(x+q)^2$. *(2 marks)*

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y=4-10x-x^2$. *(1 mark)*

(b) The curve C has equation $y=4-10x-x^2$ and the line L has equation $y=k(4x-13)$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the curve C with the line L satisfy the equation

$$x^2+2(2k+5)x-(13k+4)=0$$ *(1 mark)*

(ii) Given that the curve C and the line L intersect in two distinct points, show that

$$4k^2+33k+29>0$$ *(3 marks)*

(iii) Solve the inequality $4k^2+33k+29>0$. *(4 marks)*