RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4724:** Core Mathematics 4

Mark Scheme for January 2011

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1	**(i)**	First two terms are $1-\frac{1}{2}x$........	B1	
		Third term = $\frac{\frac{1}{2}.-\frac{1}{2}}{2}[(-x)^2 \text{ or } x^2 \text{ or } -x^2]$	M1	
		$= -\frac{1}{8}x^2$	A1 **3**	$-\frac{1}{8}x^2$ without work $\rightarrow$ M1 A1
	(ii)	Attempt to replace x by $2y-4y^2$ or $2y+4y^2$	M1	or write as $1-\left(2y-4y^2 \text{ or } 2y+4y^2\right)$
		First two terms are $1-y$	B1	
		Third term = $+\frac{3}{2}y^2$ or √ $(4b+2)y^2$	A1√ **3**	where b = cf $\left(x^2\right)$ in part (i)
			$\boxed{6}$	
2	**(i)**	$A(x-2)+B=7-2x$	M1	or $A(x-2)^2+B(x-2)=(7-2x)(x-2)$
		$A=-2$	A1	
		$B=3$	A1 **3**	
	(ii)	$\int \frac{A}{x-2}\,dx = \left(A \text{ or } \frac{1}{A}\right)\ln(x-2)$	B1	Accept ln $\lvert x-2\rvert$, ln $\lvert 2-x\rvert$, ln $(2-x)$
		$\int \frac{B}{(x-2)^2}\,dx = -\left(B \text{ or } \frac{1}{B}\right).\frac{1}{x-2}$	B1	Negative sign <u>is</u> required
		Correct f.t. of A & B; $A\ln(x-2)-\frac{B}{x-2}$	B1√	Still accept lns as before
		Using limits = $-2\ln 3+2\ln 2+\frac{1}{2}$ ISW	B1 **4**	<u>No</u> indication of ln(negative)
			$\boxed{7}$	
3	**(i)**	State/imply $\frac{d}{dx}(\sec x)=\frac{d}{dx}\left(\frac{1}{\cos x}\right)$ or $\frac{d}{dx}(\cos x)^{-1}$	B1	Not just $\sec x=\frac{1}{\cos x}$
		Attempt quotient rule or chain rule to power -1	M1	Allow $\frac{u\,dv-v\,du}{v^2}$ & wrong trig signs
		Obtain $\frac{\sin x}{\cos^2 x}$ or $-.-(\sin x)(\cos x)^{-2}$	A1	No inaccuracy allowed here
		Simplify with suff evid to **AG** e.g. $\frac{1}{\cos x}.\frac{\sin x}{\cos x}$	A1 **4**	Or vice versa. <u>Not</u> just = sec x.tan x
	(ii)	Use $\cos 2x = +/-1+/-2\cos^2 x$ or $+/-1+/-2\sin^2 x$	M1	or $\pm\left(\cos^2 x-\sin^2 x\right)$
		Correct denominator = $\sqrt{2\cos^2 x}$	A1	$\sqrt{2-2\sin^2 x}$ needs simplifying
		Evidence that $\frac{\tan x}{\cos x}$ = sec x tan x or $\int \frac{\tan x}{\cos x}\,dx = \sec x$	B1	irrespective of any const multiples
		$\frac{1}{\sqrt{2}}\sec x$ (+ c)	A1 **4**	Condone θ for x except final line
			$\boxed{8}$	

4	**(i)**	Attempt to use $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or $\frac{dy}{dt}.\frac{dt}{dx}$	M1	Not just quote formula
		$\frac{4}{2t}$ or $\frac{2}{t}$	A1 **2**	
	(ii)	Subst $t=4$ into their (i), invert & change sign	M1	
		Subst $t=4$ into (x,y) & use num grad for tgt/normal	M1	
		$y=-2x+52$ AEF CAO (no f.t.)	A1 **3**	Only the eqn of normal accepted
	(iii)	Attempt to eliminate t from the 2 given equations	M1	
		$x=2+\frac{y^2}{16}$ or $y^2=16(x-2)$ AEF ISW	A1 **2**	Mark at earliest acceptable form.
				$\boxed{7}$
5	**(i)**	Attempt to connect dx and du	M1	Including $\frac{du}{dx}=$ or $du=...dx$; not dx = du
		$5-x=4-u^2$	B1	perhaps in conjunction with next line
		Show $\int\frac{4-u^2}{2+u}.2u\,du$ reduced to $\int 4u-2u^2\,du$ **AG**	A1	In a fully satisfactory & acceptable manner
		Clear explanation of why limits change	B1	e.g. when x = 2, u = 1 <u>and</u> when x = 5, u = 2
		$\frac{4}{3}$	B1 **5**	not dependent on any of first 4 marks
	(ii)(a)	$5-x$	*B1 **1**	Accept $4-x-1=5-x$ (this is not **AG**)
	(b)	Show reduction to $2-\sqrt{x-1}$	dep*B1	
		$\int\sqrt{x-1}\,dx=\frac{2}{3}(x-1)^{\frac{3}{2}}$	B1	Indep of other marks, seen anywhere in (b)
		$\left(10-\frac{2}{3}.8\right)-\left(4-\frac{2}{3}\right)=\frac{4}{3}$ or $4\frac{2}{3}-3\frac{1}{3}=\frac{4}{3}$	B1 **3**	Working must be shown
				$\boxed{9}$
6	**(i)**	Work with correct pair of direction vectors	M1	
		Demonstrate correct <u>method</u> for finding scalar product	M1	Of <u>any</u> two 3x3 vectors rel to question
		Demonstrate correct <u>method</u> for finding modulus	M1	Of <u>any</u> vector relevant to question
		24, 24.0 (24.006363..) (degrees) 0.419 (0.41899..) (rad)	A1 **4**	Mark earliest value, allow trunc/rounding
	(ii)	Attempt to set up 3 equations	M1	Of type $3+2s=5, 3s=3+t, -2-4s=2-2t$
		Find correct values of $(s,t)=(1,0)$ or $(1,4)$ or $(5,12)$	A1	Or 2 diff values of s (or of t)
		Substitute their (s,t) into equation not used	M1	and make a relevant deduction
		<u>Correctly</u> demonstrate failure	A1 **4**	dep on all 3 prev marks
	(iii)	Subst their (s,t) from first 2 eqns into new 3rd eqn	M1	New 3rd eqn of type $a-4s=2-2t$
		$a=6$	A1 **2**	
				$\boxed{10}$

7		Attempt parts with $u = x^2+5x+7$, $dv = \sin x$	M1		as far as $f(x) +/- \int g(x)\,dx$
		1st stage = $-(x^2+5x+7)\cos x + \int (2x+5)\cos x\,dx$	A1		signs need not be amalgamated at this stage
		$\int (2x+5)\cos x\,dx = (2x+5)\sin x - \int 2\sin x\,dx$	B1		indep of previous A1 being awarded
		$= (2x+5)\sin x + 2\cos x$	B1		
		$I = -(x^2+5x+7)\cos x + (2x+5)\sin x + 2\cos x$	A1		WWW
		(Substitute $x=\pi$) $-$(Substitute $x=0$)	M1		An attempt at subst $x = 0$ must be seen
		$\pi^2+5\pi+10$ WWW **AG**	A1	**7**	
					$\boxed{7}$
8	**(i)**	$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1		
		$\frac{d}{dx}(-5xy) = (-)(5)x\frac{dy}{dx} + (-)(5)y$	M1		i.e. reasonably clear use of product rule
		LHS completely correct $4x - 5x\frac{dy}{dx} - 5y + 2y\frac{dy}{dx}(=0)$	A1		Accept "$\frac{dy}{dx} =$" provided it is not used
		Substitute $\frac{dy}{dx} = \frac{3}{8}$ or solve for $\frac{dy}{dx}$ & then equate to $\frac{3}{8}$	M1		Accuracy not required for "solve for $\frac{dy}{dx}$"
		Produce $x = 2y$ WWW **AG** **(**Converse acceptable)	A1	**5**	Expect $17x = 34y$ and/or $\frac{dy}{dx} = \frac{5y-4x}{2y-5x}$
	(ii)	Substitute $2y$ for x or $\frac{1}{2}x$ for y in curve equation	M1		
		Produce either $x^2 = 36$ or $y^2 = 9$	A1		
		AEF of $(\pm 6, \pm 3)$	A1	**3**	ISW Any correct format acceptable
					$\boxed{8}$
9	**(i)**	Attempt to sep variables in the form $\int \frac{p}{(x-8)^{1/3}}\,dx = \int q\,dt$	M1		Or invert as $\frac{dt}{dx} = \frac{r}{(x-8)^{1/3}}$; p,q,r consts
		$\int \frac{1}{(x-8)^{1/3}}\,dx = k(x-8)^{2/3}$	A1		k const
		All correct (+ c)	A1		
		For equation containing 'c'; substitute $t = 0$, $x = 72$	M1		M2 for $\int_{72}^{35} = \int_0^t$ or $\int_{35}^{72} = \int_0^t$
		Correct corresponding value of c from correct eqn	A1		
		Subst their c & $x = 35$ back into eqn	M1		
		$t = \frac{21}{8}$ or 2.63 / 2.625 [C.A.O]	A1	**7**	A2: $t = \frac{21}{8}$ or 2.63 / 2.625 WWW
	(ii)	State/imply in some way that $x = 8$ when flow stops	B1		
		Substitute $x = 8$ back into equation containing numeric 'c'	M1		
		$t = 6$	A1	**3**	
					$\boxed{10}$

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored