



**Thursday 12 June 2014 – Afternoon**

**A2 GCE MATHEMATICS**

**4727/01 Further Pure Mathematics 3**

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

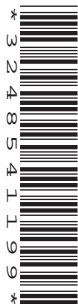
**OCR supplied materials:**

- Printed Answer Book 4727/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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## 2

- 1 (i) Find a vector equation of the line of intersection of the planes  $2x + y - z = 4$  and  $3x + 5y + 2z = 13$ . [4]  
 (ii) Find the exact distance of the point  $(2, 5, -2)$  from the plane  $2x + y - z = 4$ . [2]

- 2 Use the substitution  $u = y^2$  to find the general solution of the differential equation

$$\frac{dy}{dx} - 2y = \frac{e^x}{y}$$

for  $y$  in terms of  $x$ . [8]

- 3 (i) Solve the equation  $z^6 = 1$ , giving your answers in the form  $re^{i\theta}$ , and sketch an Argand diagram showing the positions of the roots. [4]  
 (ii) Show that  $(1 + i)^6 = -8i$ . [3]  
 (iii) Hence, or otherwise, solve the equation  $z^6 + 8i = 0$ , giving your answers in the form  $re^{i\theta}$ . [3]

- 4 The group  $G$  consists of the set  $\{1, 3, 7, 9, 11, 13, 17, 19\}$  combined under multiplication modulo 20.

(i) Find the inverse of each element. [3]

(ii) Show that  $G$  is not cyclic. [3]

(iii) Find two isomorphic subgroups of order 4 and state an isomorphism between them. [5]

- 5 Solve the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-x}$$

subject to the conditions  $y = \frac{dy}{dx} = 0$  when  $x = 0$ . [10]

- 6 The line  $l$  has equations  $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-7}{5}$ . The plane  $\Pi$  has equation  $4x - y - z = 8$ .

(i) Show that  $l$  is parallel to  $\Pi$  but does not lie in  $\Pi$ . [3]

(ii) The point  $A(1, -2, 7)$  is on  $l$ . Write down a vector equation of the line through  $A$  which is perpendicular to  $\Pi$ . Hence find the position vector of the point on  $\Pi$  which is closest to  $A$ . [4]

(iii) Hence write down a vector equation of the line in  $\Pi$  which is parallel to  $l$  and closest to it. [1]

- 7 (i) By expressing  $\sin\theta$  in terms of  $e^{i\theta}$  and  $e^{-i\theta}$ , show that

$$\sin^5\theta \equiv \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta). [4]$$

(ii) Hence solve the equation

$$\sin 5\theta + 4\sin\theta = 5\sin 3\theta$$

for  $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi$ . [4]

## 3

- 8  $G$  consists of the set of matrices of the form  $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ , where  $a$  and  $b$  are real and  $a^2 + b^2 \neq 0$ , combined under the operation of matrix multiplication.
- (i) Prove that  $G$  is a group. You may assume that matrix multiplication is associative. [6]
- (ii) Determine whether  $G$  is commutative. [2]
- (iii) Find the order of  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ . [3]

**END OF QUESTION PAPER**

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