OCR

Thursday 12 June 2014 – Afternoon

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(i)** Find a vector equation of the line of intersection of the planes $2x + y - z = 4$ and $3x + 5y + 2z = 13$. **[4]**

(ii) Find the exact distance of the point (2, 5, –2) from the plane $2x + y - z = 4$. **[2]**

2 Use the substitution $u = y^2$ to find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - 2y = \frac{\mathrm{e}^x}{y}$$

for y in terms of x. **[8]**

3 **(i)** Solve the equation $z^6 = 1$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, and sketch an Argand diagram showing the positions of the roots. **[4]**

(ii) Show that $(1 + \mathrm{i})^6 = -8\mathrm{i}$. **[3]**

(iii) Hence, or otherwise, solve the equation $z^6 + 8\mathrm{i} = 0$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$. **[3]**

4 The group G consists of the set {1, 3, 7, 9, 11, 13, 17, 19} combined under multiplication modulo 20.

(i) Find the inverse of each element. **[3]**

(ii) Show that G is not cyclic. **[3]**

(iii) Find two isomorphic subgroups of order 4 and state an isomorphism between them. **[5]**

5 Solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 5\frac{\mathrm{d}y}{\mathrm{d}x} + 6y = \mathrm{e}^{-x}$$

subject to the conditions $y = \dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = 0$. **[10]**

6 The line l has equations $\dfrac{x-1}{2} = \dfrac{y+2}{3} = \dfrac{z-7}{5}$. The plane Π has equation $4x - y - z = 8$.

(i) Show that l is parallel to Π but does not lie in Π. **[3]**

(ii) The point A (1, –2, 7) is on l. Write down a vector equation of the line through A which is perpendicular to Π. Hence find the position vector of the point on Π which is closest to A. **[4]**

(iii) Hence write down a vector equation of the line in Π which is parallel to l and closest to it. **[1]**

7 **(i)** By expressing $\sin\theta$ in terms of $\mathrm{e}^{\mathrm{i}\theta}$ and $\mathrm{e}^{-\mathrm{i}\theta}$, show that

$$\sin^5\theta \equiv \tfrac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta).$$ **[4]**

(ii) Hence solve the equation

$$\sin 5\theta + 4\sin\theta = 5\sin 3\theta$$

for $-\tfrac{1}{2}\pi \leqslant \theta \leqslant \tfrac{1}{2}\pi$. **[4]**

8 G consists of the set of matrices of the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, where a and b are real and $a^2 + b^2 \neq 0$, combined under the operation of matrix multiplication.

(i) Prove that G is a group. You may assume that matrix multiplication is associative. **[6]**

(ii) Determine whether G is commutative. **[2]**

(iii) Find the order of $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. **[3]**

END OF QUESTION PAPER

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE