WMA02/01: Core Mathematics C34

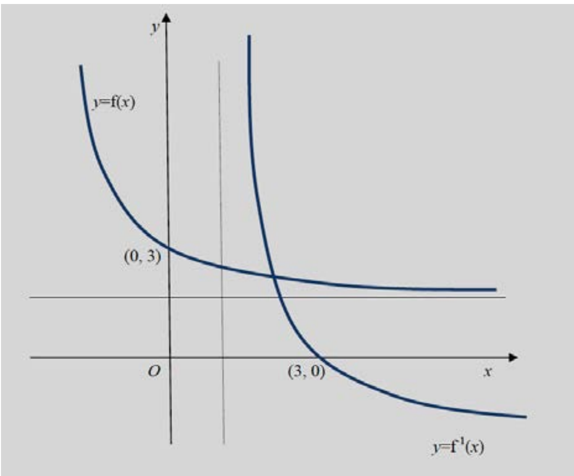
Question Number	Scheme	Marks
1. (a)	$R = 13$	B1
	$\tan\alpha = \frac{12}{5} \Rightarrow \alpha = 67.38°$	M1 A1
		(3)
(b)	$13\cos(2\theta + 67.4°) = 10 \Rightarrow \cos(2\theta + 67.4°) = \frac{10}{13}$	M1
	$2\theta + 67.38° = 39.715°, (320.285°, 399.715°)$	A1
	$\theta = 126.5°$	A1
	$\theta = 166.2°$	M1 A1
		(5) **(8 marks)**
2. (a)		

x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	**1.844321332**	**4.810477381**	8.87207	0

Question Number	Scheme	Marks
	awrt 1.84432	B1
	awrt 4.81048 or 4.81047	B1
		(2)
(b)	$\frac{1}{2} \times \frac{\pi}{4}$ or $\frac{\pi}{8}$	B1
	Area $\approx \frac{1}{2} \times \frac{\pi}{4}$;$\times \underline{\left\{0 + 2\left(1.84432 + 4.81048 + 8.87207\right) + 0\right\}}$	M1
	<u>12.1948</u>	A1
		(3)
(c)	Uses $vu' + uv'$ $\quad \frac{dy}{dx} = e^x \times \frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x) + e^x(\sin x)^{\frac{1}{2}}$	M1 A1 A1
	$\frac{dy}{dx} = 0 \Rightarrow e^x \times \frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x) + e^x(\sin x)^{\frac{1}{2}} = 0$	
	$\cos x = -2\sin x$	M1
	$\tan x = -\frac{1}{2} \Rightarrow x = 2.68$	M1 A1
		(6) **(11 marks)**

Question Number	Scheme	Marks
3.	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = -\sin x$	B1
	$\int \mathrm{e}^{\cos x+1}\sin x\,\mathrm{d}x = -\int \mathrm{e}^{u}\,\mathrm{d}u$	M1 A1
	$= -\mathrm{e}^{u}(+c)$ ft on sign error	A1
	$= -\mathrm{e}^{(\cos x+1)}(+c)$	
	$\left[-\mathrm{e}^{(\cos x+1)}\right]_0^{\frac{\pi}{2}} = (-\mathrm{e}) - (-\mathrm{e}^2) = \mathrm{e}(\mathrm{e}-1)$ cso	M1 A1*
		(6 marks)
4. (a)	$(2-3x)^{-2} = 2^{-2}\left(1-\dfrac{3x}{2}\right)^{-2}$	B1
	$\left(1-\dfrac{3x}{2}\right)^{-2} = 1+(-2)\left(-\dfrac{3x}{2}\right)+\dfrac{(-2)(-3)}{2\times1}\left(-\dfrac{3x}{2}\right)^2+\dfrac{(-2)(-3)(-4)}{3\times2\times1}\left(-\dfrac{3x}{2}\right)^3+\ldots$	M1 A1
	$= 1+3x+\dfrac{27}{4}x^2+\dfrac{27}{2}x^3+..$	
	$(2-3x)^{-2} = \dfrac{1}{4}+\dfrac{3}{4}x+\dfrac{27}{16}x^2+\dfrac{27}{8}x^3+..$	M1 A1
		(5)
(b)	$\mathrm{f}(x) = (a+bx)\left(\dfrac{1}{4}+\dfrac{3}{4}x+\dfrac{27}{16}x^2+\dfrac{27}{8}x^3+..\right)$	
	Coefficient of x: $\dfrac{3a}{4}+\dfrac{b}{4}=0$ $(3a+b=0)$	M1
	Coefficient of x^2: $\dfrac{27a}{16}+\dfrac{3b}{4}=\dfrac{9}{16}$ $(9a+4b=3)$	M1
	A1 (either correct)	A1
	Leading to $a=-1, b=3$	dM1 A1
		(5)
(c)	Coefficient of x^3: $\dfrac{27a}{8}+\dfrac{27b}{16}=\dfrac{27}{8}\times-1+\dfrac{27}{16}\times3$	M1 A1ft
	$=\dfrac{27}{16}=\left(1\dfrac{11}{16}\right)$ cao	A1
		(3) **(13 marks)**

WMA02/01: Core Mathematics C34

Question Number	Scheme	Marks
5. (a)	$\mathrm{fg}(x) = \mathrm{e}^{-2\ln x} + 2$,	M1
	$= \mathrm{e}^{\ln x^{-2}} + 2 = x^{-2} + 2 = \left(\dfrac{1}{x^2} + 2\right)$	M1 A1
		(3)
(b)	$\mathrm{e}^{-(2x+3)} + 2 = 6 \Rightarrow \mathrm{e}^{-(2x+3)} = 4$	M1 A1
	$\Rightarrow -(2x+3) = \ln 4$	
	$\Rightarrow x = \dfrac{-3 - \ln 4}{2}$	M1 A1
		(4)
(c)	Let $y = \mathrm{e}^{-x} + 2 \Rightarrow y - 2 = \mathrm{e}^{-x} \Rightarrow \ln(y-2) = -x$	
	$\Rightarrow x = -\ln(y-2)$	M1
	$\mathrm{f}^{-1}(x) = -\ln(x-2), \quad x > 2.$	A1 B1
		(3)
(d)		
	Shape for $\mathrm{f}(x)$	B1
	$(0, 3)$	B1
	Shape for $\mathrm{f}^{-1}(x)$	B1
	$(3, 0)$	B1
		(4)
		(14 marks)

Question Number	Scheme	Marks
6. (a)	Differentiating implicitly to obtain $\pm ay^2\dfrac{dy}{dx}$ and/or $\pm bx^2\dfrac{dy}{dx}$	M1
	$48y^2\dfrac{dy}{dx} + \ ... \ -54\ ...$	A1
	$9x^2y \to 9x^2\dfrac{dy}{dx} + 18xy$ or equivalent	B1
	$\left(48y^2+9x^2\right)\dfrac{dy}{dx} + 18xy - 54 = 0$	M1
	$\dfrac{dy}{dx} = \dfrac{54-18xy}{48y^2+9x^2} \quad \left(= \dfrac{18-6xy}{16y^2+3x^2}\right)$	A1
		(5)
(b)	$18 - 6xy = 0$	M1
	Using $x = \dfrac{3}{y}$ or $y = \dfrac{3}{x}$	
	$16y^3 + 9\left(\dfrac{3}{y}\right)^2 y - 54\left(\dfrac{3}{y}\right) = 0$ or $16\left(\dfrac{3}{x}\right)^3 + 9x^2\left(\dfrac{3}{x}\right) - 54x = 0$	M1
	Leading to	
	$16y^4 + 81 - 162 = 0$ or $16 + x^4 - 2x^4 = 0$	M1
	$y^4 = \dfrac{81}{16}$ or $x^4 = 16$	
	$y = \dfrac{3}{2}, -\dfrac{3}{2}$ or $x = 2, -2$	A1, A1
	Subs either of their values into $xy = 3$ to obtain a value of other variable.	M1
	$\left(2, \dfrac{3}{2}\right), \left(-2, -\dfrac{3}{2}\right)$ both	A1
		(7) **(12 marks)**

Question Number	Scheme	Marks
7. (a)	$\cot x - \cot 2x \equiv \dfrac{\cos x}{\sin x} - \dfrac{\cos 2x}{\sin 2x}$	B1
	$\equiv \dfrac{\sin 2x \cos x - \cos 2x \sin x}{\sin x \sin 2x}$	M1
	$\equiv \dfrac{\sin(2x - x)}{\sin x \sin 2x}$	M1
	$\equiv \dfrac{\sin x}{\sin x \sin 2x} \equiv \dfrac{1}{\sin 2x} \equiv \operatorname{cosec} 2x$	M1 A1*
		(5)
(b)	$2x = 3\theta + \dfrac{\pi}{3} \Rightarrow x = 1.5\theta + \dfrac{\pi}{6}$	B1
	$\cot\left(1.5\theta + \dfrac{\pi}{6}\right) = \dfrac{1}{\sqrt{3}} \Rightarrow \tan\left(1.5\theta + \dfrac{\pi}{6}\right) = \sqrt{3}$	M1
	$\left(1.5\theta + \dfrac{\pi}{6}\right) = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$	M1
	$\theta = \dfrac{\pi}{9}, \dfrac{7\pi}{9}$	A1, A1
		(5) **(10 marks)**

Question Number	Scheme	Marks
8. (a)	$\dfrac{2}{x+2}+\dfrac{4}{x^2+5}-\dfrac{18}{(x+2)(x^2+5)}=\dfrac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$	M1 A1
	$=\dfrac{2x(x+2)}{(x+2)(x^2+5)}$	M1
	$=\dfrac{2x}{(x^2+5)}$	A1*
		(4)
(b)	$\text{h}'(x)=\dfrac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$	M1 A1
	$\text{h}'(x)=\dfrac{10-2x^2}{(x^2+5)^2}$	A1
		(3)
(c)	Maximum occurs when $\text{h}'(x)=0 \Rightarrow 10-2x^2=0 \Rightarrow x=..$	M1
	$\Rightarrow x=\sqrt{5}$	A1
	When $x=\sqrt{5} \Rightarrow \text{h}(x)=\dfrac{\sqrt{5}}{5}$	M1 A1
	Range of h(x) is $0 \leqslant h(x) \leqslant \dfrac{\sqrt{5}}{5}$	A1ft
		(5) **(12 marks)**

Question Number	Scheme	Marks
9. (a)	Equate **j** components $3+2\lambda=9 \Rightarrow \lambda=3$	M1 A1
	Leading to $C=(5,9,-1)$	A1
		(3)
(b)	Choosing correct directions or finding $\overrightarrow{AC}$ and $\overrightarrow{BC}$	M1
	$\begin{pmatrix}1\\2\\1\end{pmatrix}.\begin{pmatrix}5\\0\\2\end{pmatrix}=5+2=\sqrt{6}\times\sqrt{29}\times\cos\angle ABC$ Use of scalar product.	M1 A1
	$\angle ACB=57.95°$	A1
		(4)
(c)	$A=(2,3,-4)$ $B=(-5,9,-5)$	
	$\overrightarrow{AC}=\begin{pmatrix}3\\6\\3\end{pmatrix}$ AND $\overrightarrow{BC}=\begin{pmatrix}10\\0\\4\end{pmatrix}$	
	$AC^2=3^2+6^2+3^2=\left(3\sqrt{6}\right)$ $BC^2=10^2+4^2=\left(2\sqrt{29}\right)$	M1 A1 A1
	Area triangle $ABC=\frac{1}{2}AC\,BC\sin\angle ACB=\frac{1}{2}\times'3\sqrt{6}'\times'2\sqrt{29}'\times\sin 57.95°$	M1
	$=33.5$	A1
		(5)
		(12 marks)

Question Number	Scheme		Marks
10. (a)	$\tan\theta = \sqrt{3}$ *or* $\sin\theta = \dfrac{\sqrt{3}}{2}$		M1
	$\theta = \dfrac{\pi}{3}$	awrt 1.05	A1
			(2)
(b)	$\dfrac{dx}{d\theta} = \sec^2\theta$, $\dfrac{dy}{d\theta} = \cos\theta$		
	$\dfrac{dy}{dx} = \dfrac{\cos\theta}{\sec^2\theta} \quad \left(=\cos^3\theta\right)$		M1 A1
	At P, $m = \cos^3\left(\dfrac{\pi}{3}\right) = \dfrac{1}{8}$	Can be implied.	A1
	Using $mm' = -1$, $m' = -8$		M1
	For normal $y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$		dM1
	At Q, $y = 0$ $\quad -\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$		
	leading to $x = \frac{17}{16}\sqrt{3} \quad \left(k = \frac{17}{16}\right)$	1.0625	A1
			(6)
(c)	$\int y^2\, dx = \int y^2 \dfrac{dx}{d\theta} d\theta = \int \sin^2\theta \sec^2\theta\, d\theta$		M1 A1
	$= \int \tan^2\theta\, d\theta$		A1
	$= \int \left(\sec^2\theta - 1\right) d\theta$		dM1
	$= \tan\theta - \theta \quad (+C)$		A1
	$V = \pi\int_0^{\frac{\pi}{3}} y^2\, dx = \left[\tan\theta - \theta\right]_0^{\frac{\pi}{3}} = \pi\left[\left(\sqrt{3} - \frac{\pi}{3}\right) - (0 - 0)\right]$		dM1
	$= \sqrt{3}\pi - \frac{1}{3}\pi^2 \quad \left(p = 1, q = -\frac{1}{3}\right)$		A1
			(7) **(15 marks)**

Question Number	Scheme	Marks
11. (a)	$\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int \frac{1}{15}\,\mathrm{d}t$	B1
	$1 = A(5-P) + BP$	M1
	$A = \frac{1}{5},\ B = \frac{1}{5}$	A1
	giving $\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int \frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}\mathrm{d}P$	A1
	Hence $\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int \frac{1}{15}\,\mathrm{d}t$	
	$\Rightarrow \frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \ \ (+c)$	M1 A1ft
	$\{t = 0, P = 1 \Rightarrow\}\ \ \frac{1}{5}\ln 1 - \frac{1}{5}\ln(4) = 0 + c \ \ \left\{\Rightarrow\ c = -\frac{1}{5}\ln 4\right\}$	dM1
	eg: $\frac{1}{5}\ln\left(\frac{P}{5-P}\right) = \frac{1}{15}t - \frac{1}{5}\ln 4$	
	$\ln\left(\frac{4P}{5-P}\right) = \frac{1}{3}t$ Using any of the subtraction (or addition) laws for logarithms CORRECTLY.	dM1
	eg: $\frac{4P}{5-P} = \mathrm{e}^{\frac{1}{3}t}$ or eg: $\frac{5-P}{4P} = \mathrm{e}^{-\frac{1}{3}t}$ Eliminate ln's correctly.	dM1
	gives $4P = 5\mathrm{e}^{\frac{1}{3}t} - P\mathrm{e}^{\frac{1}{3}t} \Rightarrow P(4 + \mathrm{e}^{\frac{1}{3}t}) = 5\mathrm{e}^{\frac{1}{3}t}$	
	$P = \frac{5\mathrm{e}^{\frac{1}{3}t}}{(4 + \mathrm{e}^{\frac{1}{3}t})}$ $\left\{\frac{(\div\, \mathrm{e}^{\frac{1}{3}t})}{(\div\, \mathrm{e}^{\frac{1}{3}t})}\right\}$ Make P the subject.	dM1
	$P = \frac{5}{(1 + 4\mathrm{e}^{-\frac{1}{3}t})}$ or $P = \frac{25}{(5 + 20\mathrm{e}^{-\frac{1}{3}t})}$ etc.	A1
	Note that the 'dM' marks are dependent upon the first two M marks.	(11)
(b)	$1 + 4\mathrm{e}^{-\frac{1}{3}t} > 1 \ \Rightarrow\ P < 5$. So population cannot exceed 5000	B1 (1)
		(12 marks)