

# OCR

Oxford Cambridge and RSA

## Tuesday 9 June 2015 – Morning

### A2 GCE MATHEMATICS

**4727/01** Further Pure Mathematics 3**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4727/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

## 2

- 1 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = \sin x. \quad [8]$$

- 2 The elements of a group  $G$  are polynomials of the form  $a + bx + cx^2$ , where  $a, b, c \in \{0, 1, 2, 3, 4\}$ . The group operation is addition, where the coefficients are added modulo 5.

(i) State the identity element. [1]

(ii) State the inverse of  $3 + 2x + x^2$ . [2]

(iii) State the order of  $G$ . [1]

The proper subgroup  $H$  contains  $2 + x$  and  $1 + x$ .

(iv) Find the order of  $H$ , justifying your answer. [4]

- 3 The plane  $\Pi$  passes through the points  $(1, 2, 1)$ ,  $(2, 3, 6)$  and  $(4, -1, 2)$ .

(i) Find a cartesian equation of the plane  $\Pi$ . [5]

The line  $l$  has equation  $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$ .

(ii) Find the coordinates of the point of intersection of  $\Pi$  and  $l$ . [3]

(iii) Find the acute angle between  $\Pi$  and  $l$ . [3]

- 4 In an Argand diagram, the complex numbers  $0$ ,  $z$  and  $ze^{\frac{1}{6}i\pi}$  are represented by the points  $O$ ,  $A$  and  $B$  respectively.

(i) Sketch a possible Argand diagram showing the triangle  $OAB$ . Show that the triangle is isosceles and state the size of angle  $AOB$ . [4]

The complex numbers  $1 + i$  and  $5 + 2i$  are represented by the points  $C$  and  $D$  respectively. The complex number  $w$  is represented by the point  $E$ , such that  $CD = CE$  and angle  $DCE = \frac{1}{6}\pi$ .

(ii) Calculate the possible values of  $w$ , giving your answers exactly in the form  $a + bi$ . [5]

- 5 Find the particular solution of the differential equation

$$x\frac{dy}{dx} + 3y = x^2 + x$$

for which  $y = 1$  when  $x = 1$ , giving  $y$  in terms of  $x$ . [8]

3

6 Find the shortest distance between the lines with equations

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-1} \quad \text{and} \quad \frac{x-3}{4} = \frac{y-1}{-2} = \frac{z+1}{3}. \quad [7]$$

7 (i) Use de Moivre's theorem to show that  $\tan 4\theta \equiv \frac{4 \tan \theta - 4 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta}$ . [4]

(ii) Hence find the exact roots of  $t^4 + 4\sqrt{3}t^3 - 6t^2 - 4\sqrt{3}t + 1 = 0$ . [5]

8 Let  $G$  be any multiplicative group.  $H$  is a subset of  $G$ .  $H$  consists of all elements  $h$  such that  $hg = gh$  for every element  $g$  in  $G$ .

(i) Prove that  $H$  is a subgroup of  $G$ . [8]

Now consider the case where  $G$  is given by the following table:

	<i>e</i>	<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	<i>t</i>
<i>e</i>	<i>e</i>	<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	<i>t</i>
<i>p</i>	<i>p</i>	<i>q</i>	<i>e</i>	<i>s</i>	<i>t</i>	<i>r</i>
<i>q</i>	<i>q</i>	<i>e</i>	<i>p</i>	<i>t</i>	<i>r</i>	<i>s</i>
<i>r</i>	<i>r</i>	<i>t</i>	<i>s</i>	<i>e</i>	<i>q</i>	<i>p</i>
<i>s</i>	<i>s</i>	<i>r</i>	<i>t</i>	<i>p</i>	<i>e</i>	<i>q</i>
<i>t</i>	<i>t</i>	<i>s</i>	<i>r</i>	<i>q</i>	<i>p</i>	<i>e</i>

(ii) Show that  $H$  consists of just the identity element. [4]

**END OF QUESTION PAPER**

---

# OCR

Oxford Cambridge and RSA

## Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.