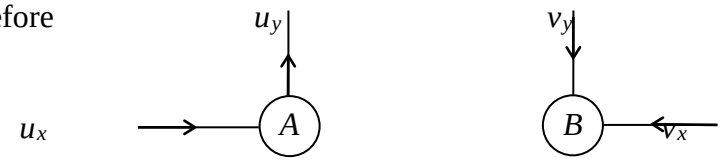
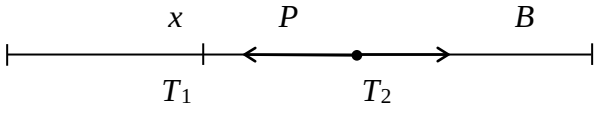
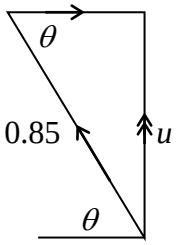
EDEXCEL 6680 MECHANICS M4 JANUARY 2004 MARK SCHEME

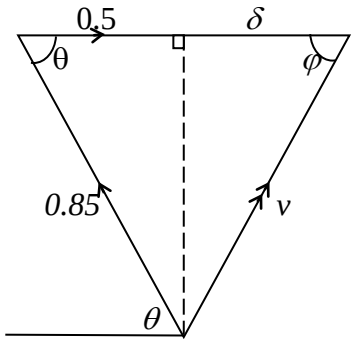
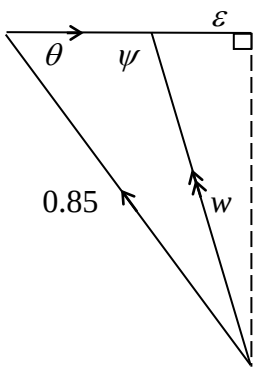
Question Number	Scheme	Marks
1.	N2L $\quad -2v = 3a$	M1
	$-2v = 3v\dfrac{\mathrm{d}v}{\mathrm{d}s}$	A1
	$s = -\dfrac{3}{2}v\ (+c)$ or $v = -\dfrac{2}{3}s\ (+c)$ cancelling v and integrating	M1
	$s = 0, v = 5 \Rightarrow c = \dfrac{15}{2}$ or $s = \left[-\dfrac{3}{2}v\right]_5^2$	M1
	Distance travelled is $4.5\,\mathrm{m}$	A1 **(5 marks)**
2.	(a) Before 	
	A: $\uparrow$ $u_y = 2.5\sin\alpha = 2.5\times\frac{4}{5} = 2\ \left(\mathrm{ms}^{-1}\right)$ either	M1
	$\rightarrow$ $u_x = 2.5\cos\alpha = 2.5\times\frac{3}{5} = 1.5\ \left(\mathrm{ms}^{-1}\right)$ both	A1
	B: $\downarrow$ $v_y = 1.3\sin\beta = 1.3\times\frac{12}{13} = 1.2\ \left(\mathrm{ms}^{-1}\right)$ either	M1
	$\leftarrow$ $v_x = 1.3\cos\beta = 1.3\times\frac{5}{13} = 0.5\ \left(\mathrm{ms}^{-1}\right)$ both	A1 (4)
	(b) After 	
	LM $\quad 2x + w = 3 - 0.5 \quad (= 2.5)$	M1 A1 ft
	NEL $\quad w - x = \frac{1}{2}\times 2 \quad (=1)$	M1 A1 ft
	Solving $\quad x = 0.5,\ y = 1.5$ M1 solving for either	M1 A1
	Speed of A is $\sqrt{\left(2^2 + 0.5^2\right)} = \sqrt{4.25} \approx 2.1\ \left(\mathrm{ms}^{-1}\right)$ M1 either	M1 A1
	Speed of B is $\sqrt{\left(1.2^2 + 1.5^2\right)} = \sqrt{3.69} \approx 1.9\ \left(\mathrm{ms}^{-1}\right)$	A1 (9) **(13 marks)**
	Note: *Not* 1 *d.p. loses maximum of one mark*	

Question Number	Scheme	Marks
3.	(a) $AP = s - AD - DE$ $= s - L - 2L\sin\theta$	M1 A1 (2)
	(b) $V(\theta) = 2\times 2mg\times L\cos\theta + ...$ $= \quad ... \quad + mg(2L\cos\theta - AP)$ $= 4mgL\cos\theta + mg\left(2L\cos\theta + 2L\sin\theta\right)(+C)$ $= 2mgL\left(3\cos\theta + \sin\theta\right) +$ constant (*) cso	B1 M1 M1 A1 (4)
	(c) $V`(\theta) = 2mgL\ (-3\sin\theta + \cos\theta)$ $= 0$ $\tan\theta = \frac{1}{3}$ $\theta \approx 18°$ awrt 18°, 0.32^c	M1 M1 A1 A1 (4)
	(d) $V''(\theta) = 2mgL\left(-3\cos\theta - \sin\theta\right)$ $\left(V''\left(\arctan\frac{1}{3}\right) = -2\sqrt{10}mgL\right)$ $V''(\theta) < 0$, for any acute θ Equilibrium is <u>unstable</u> ft any acute θ	M1 A1 M1 A1 ft (4) **(14 marks)**

Question Number	Scheme	Marks
4.	(a) 	
	HL $T_1 = \dfrac{2mk^2L(0.5L+x)}{L}$ either	M1
	HL $T_2 = \dfrac{2mk^2L(0.5L-x)}{L}$ both	A1
	N2L $T_2 - T_1 - 2mk\dfrac{dx}{dt}, = m\dfrac{d^2x}{dt^2}$	M1 A1, A1
	$4mk^2x - 2mk\dfrac{dx}{dt} = m\dfrac{d^2x}{dt^2}$	
	$\dfrac{d^2x}{dt^2} + 2k\dfrac{dx}{dt} + 4k^2x = 0$ * cso	A1 (6)
	(b) $m^2 + 2km + 4m^2 = 0$ ae	M1
	$m = -k \pm k\sqrt{3}\mathrm{i}$	M1
	$x = \mathrm{e}^{-kt}\left(A\cos\sqrt{3}kt + B\sin\sqrt{3}kt\right)$ oe	A1
	$t = 0, x = \dfrac{L}{2} \Rightarrow A = \dfrac{L}{2}$	B1
	$\dot{x} = -k\,\mathrm{e}^{-kt}\left(A\cos\sqrt{3}kt + B\sin\sqrt{3}kt\right)$	
	$+\sqrt{3}k\,\mathrm{e}^{-kt}\left(-A\sin\sqrt{3}kt + B\cos\sqrt{3}kt\right)$	M1
	$t = 0, \dot{x} = 0 \Rightarrow 0 = -kA + \sqrt{3}kB$	M1
	$B = \dfrac{1}{\sqrt{3}}A = \dfrac{L}{2\sqrt{3}}$	A1
	$AP = 1.5L + \dfrac{L}{2\sqrt{3}}\mathrm{e}^{-kt}\left(\sqrt{3}\cos\sqrt{3}kt + \sin\sqrt{3}kt\right)$ oe	A1 (8)
		(14 marks)
	Alternatives forms of the answer are given on the next page	

Question Number	Scheme	Marks
4.	(b) *Alternative form of the General Solution* As before	M1 M1
	$x = A\mathrm{e}^{-kt}\cos\left(\sqrt{3}kt - \varepsilon\right)$	A1
	$t = 0, x = \frac{L}{2} \Rightarrow \frac{L}{2} = A\cos(-\varepsilon)\ \left(= A\cos\varepsilon\right)$	B1
	$\dot{x} = -kA\mathrm{e}^{-kt}\cos\left(\sqrt{3}kt - \varepsilon\right) - \sqrt{3}kA\mathrm{e}^{-kt}\sin\left(\sqrt{3}kt - \varepsilon\right)$	M1
	$t = 0, \dot{x} = 0 \Rightarrow 0 = -kA\cos\varepsilon - \sqrt{3}kA\sin(-\varepsilon)$	M1
	Leading to $\tan\varepsilon = \frac{1}{\sqrt{3}} \Rightarrow \varepsilon = \frac{\pi}{6}$ and $A = \frac{L}{\sqrt{3}}$ both	A1
	$AP = 1.5L + \frac{L}{\sqrt{3}}\mathrm{e}^{-kt}\cos\left(\sqrt{3}kt - \frac{\pi}{6}\right)$	A1 (8)
	Note: *Another possible trig form is* $\sin\left(\sqrt{3}kt + \frac{\pi}{3}\right)$	
5.	(a)Before After 	
	$\rightarrow$ LM $600u = 800x$	M1 A1
	$\rightarrow$ NEL $x = eu$	M1 A1
	$e = 0.75$	A1 (5)
	(b) Van N2L $-500 = 800a$	M1
	$0^2 = x^2 - 2\times 0.625\times 45,\quad x^2 = 56.25\quad (x = 7.5)$	M1, A1
	Car N2L $-300 = 600a$	M1
	$0^2 = v^2 - 2\times 0.5\times 21,\quad v^2 = 21$	M1, A1
	From (a) NEL $u = \frac{4}{3}\times 7.5 = 10$	M1
	$V^2 = 10^2 + 21, \Rightarrow V = 11\ \left(\mathrm{ms}^{-1}\right)$ cao	M1, A1 (9)
		(14 marks)

Question Number	Scheme	Marks
6.	 (a) Vector ! or ←	M1
	$\cos\theta = \frac{0.4}{0.85}$	M1
	$\theta \approx 61.9°$ awrt $62°$	A1 (3)
	(b) $u = \sqrt{(0.85^2 - 0.4^2)}$ or $u = 0.85\sin\theta$	M1
	$t = \frac{60}{u} = \frac{60}{0.75} = 80 \ (\text{s})$ cao	M1 A1 (3)
	(c) $\mathbf{v}_{N\,\text{rel}\,W} = -0.4\mathbf{i}\ (+0.75\mathbf{j})$ Allow for $\pm 0.4\mathbf{i}$	M1
	$\mathbf{v}_N = \mathbf{v}_{N\,\text{rel}\,W} + 0.5\mathbf{i} = 0.1\mathbf{i} + (0.75\mathbf{j})$ $0.1\mathbf{i}$	A1
	$t = \frac{40}{0.75} = \frac{160}{3}$	M1
	$\delta = 0.1 \times \frac{160}{3} = \frac{16}{3}$ awrt 5.3	M1 A1 (5)
	(d) As in (c) $\mathbf{v}_N = -0.2\mathbf{i} + 0.75\mathbf{j}$ $\pm 0.2\mathbf{i}$	M1
	$t = \frac{20}{0.75} = \frac{80}{3}$	M1
	$\delta = 0.2 \times \frac{80}{3} = \frac{16}{3}$	M1
	Hence *N* lands at *D* cso	A1 (4) **(15 marks)**
	Notes: 1. *In* (c) *and* (d), *the candidate can take components without using vectors. Mark as vector method.* 2. *After the first line in* (d), *the result is clear by proportion. Allow as long as some explanation given.* 3. $\cos\theta = \frac{8}{17} = 0.4705..., \sin\theta = \frac{15}{17} = 0.8823...$ 4. *Alternatives to* (c) *and* (d), *using vector triangles are given on the next page.*	

Question Number	Scheme	Marks
6.	*Alternatives to* (c) *and* (d)	
(c)		
	$v^2 = 0.5^2 + 0.85^2 - 2 \times 0.5 \times 0.85 \times \cos\theta$	
	$= 0.5725$ $\left(v = \frac{\sqrt{229}}{20} \approx 82.4°\right)$	M1
	$\frac{\sin\varphi}{0.85} = \frac{\sin\theta}{v}$	M1
	$\sin\varphi = \frac{15}{\sqrt{229}}$ $(\approx 0.9912;\ \varphi \approx 82.4°)$	A1
	$\frac{\delta}{40} = \cot\varphi;\quad \delta = 40 \times \frac{2}{5} = \frac{16}{3}$ awrt 5. 3	M1 A1 (5)
(d)	$w^2 = 0.2^2 + 0.85^2 - 2 \times 0.2 \times 0.85 \times \cos\theta$	
		
	$= 0.6025$ $\left(w = \frac{\sqrt{241}}{20} \approx 0.7762...\right)$	M1
	$\frac{\sin\psi}{0.85} = \frac{\sin\theta}{w}$	
	$\sin\psi = \frac{15}{\sqrt{241}}$ $(\approx 0.9662;\ \psi \approx 104.9°$	M1
	$\psi = 75.1°$ gains M1	
	$\frac{\varepsilon}{20} = \cot(180° - \psi) = \frac{4}{15}$	
	$\varepsilon = \frac{16}{3} = \delta$	M1
	Hence N lands at D cso	A1 (4)
	Note: Exact working is needed for final A1 *but all previous marks in* (c) *and* (d) *may be gained by approximate working*.	