



General Certificate of Education
Advanced Subsidiary Examination
June 2011

Mathematics

MPC2

Unit Pure Core 2

Wednesday 18 May 2011 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

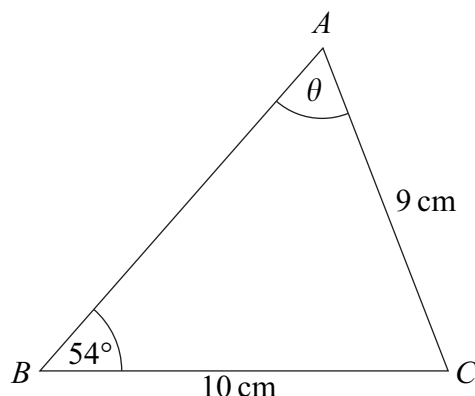
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

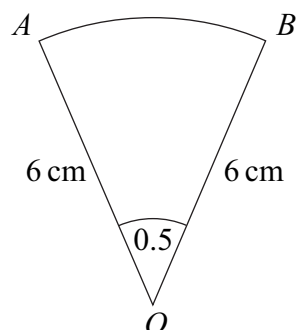
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- 1 The triangle ABC , shown in the diagram, is such that $AC = 9$ cm, $BC = 10$ cm, angle $ABC = 54^\circ$ and the acute angle $BAC = \theta$.



- (a) Show that $\theta = 64^\circ$, correct to the nearest degree. (3 marks)
- (b) Calculate the area of triangle ABC , giving your answer to the nearest square centimetre. (3 marks)
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- 2 The diagram shows a sector OAB of a circle with centre O .



The radius of the circle is 6 cm and the angle $AOB = 0.5$ radians.

- (a) Find the area of the sector OAB . (2 marks)
- (b) (i) Find the length of the arc AB . (2 marks)
- (ii) Hence show that

the perimeter of the sector $OAB = k \times$ the length of the arc AB

where k is an integer.

(2 marks)



3 (a) The expression $(2 + x^2)^3$ can be written in the form

$$8 + px^2 + qx^4 + x^6$$

Show that $p = 12$ and find the value of the integer q . (3 marks)

(b) (i) Hence find $\int \frac{(2 + x^2)^3}{x^4} dx$. (5 marks)

(ii) Hence find the exact value of $\int_1^2 \frac{(2 + x^2)^3}{x^4} dx$. (2 marks)

4 (a) Sketch the curve with equation $y = 4^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. (2 marks)

(b) Describe the geometrical transformation that maps the graph of $y = 4^x$ onto the graph of $y = 4^x - 5$. (2 marks)

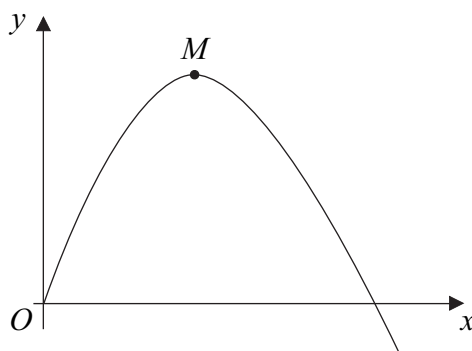
(c) (i) Use the substitution $Y = 2^x$ to show that the equation $4^x - 2^{x+2} - 5 = 0$ can be written as $Y^2 - 4Y - 5 = 0$. (2 marks)

(ii) Hence show that the equation $4^x - 2^{x+2} - 5 = 0$ has only one real solution. Use logarithms to find this solution, giving your answer to three decimal places. (4 marks)



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- 5 The diagram shows part of a curve with a maximum point M .



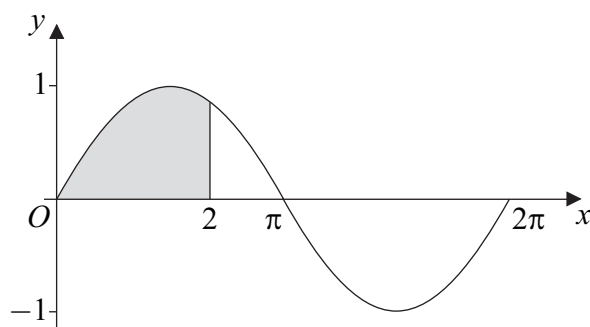
The curve is defined for $x \geq 0$ by the equation

$$y = 6x - 2x^{\frac{3}{2}}$$

- (a) Find $\frac{dy}{dx}$. (3 marks)
- (b) (i) Hence find the coordinates of the maximum point M . (3 marks)
- (ii) Write down the equation of the normal to the curve at M . (1 mark)
- (c) The point $P\left(\frac{9}{4}, \frac{27}{4}\right)$ lies on the curve.
- (i) Find an equation of the normal to the curve at the point P , giving your answer in the form $ax + by = c$, where a , b and c are positive integers. (4 marks)
- (ii) The normals to the curve at the points M and P intersect at the point R . Find the coordinates of R . (2 marks)



- 6 A curve C , defined for $0 \leq x \leq 2\pi$ by the equation $y = \sin x$, where x is in radians, is sketched below. The region bounded by the curve C , the x -axis from 0 to 2 and the line $x = 2$ is shaded.



- (a) The area of the shaded region is given by $\int_0^2 \sin x \, dx$, where x is in radians.

Use the trapezium rule with five ordinates (four strips) to find an approximate value for the area of the shaded region, giving your answer to three significant figures.

(4 marks)

- (b) Describe the geometrical transformation that maps the graph of $y = \sin x$ onto the graph of $y = 2 \sin x$. (2 marks)
- (c) Use a trigonometrical identity to solve the equation

$$2 \sin x = \cos x$$

in the interval $0 \leq x \leq 2\pi$, giving your solutions in radians to three significant figures.

(4 marks)

- 7 The n th term of a sequence is u_n . The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first two terms of the sequence are given by $u_1 = 60$ and $u_2 = 48$.

The limit of u_n as n tends to infinity is 12.

- (a) Show that $p = \frac{3}{4}$ and find the value of q . (5 marks)
- (b) Find the value of u_3 . (1 mark)

Turn over ►



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8 Prove that, for all values of x , the value of the expression

$$(3 \sin x + \cos x)^2 + (\sin x - 3 \cos x)^2$$

is an integer and state its value.

(4 marks)

9 The first term of a geometric series is 12 and the common ratio of the series is $\frac{3}{8}$.

(a) Find the sum to infinity of the series. (2 marks)

(b) Show that the sixth term of the series can be written in the form $\frac{3^6}{2^{13}}$. (3 marks)

(c) The n th term of the series is u_n .

(i) Write down an expression for u_n in terms of n . (1 mark)

(ii) Hence show that

$$\log_a u_n = n \log_a 3 - (3n - 5) \log_a 2 \quad (4 \text{ marks})$$

END OF QUESTIONS

