



**ADVANCED GCE UNIT
MATHEMATICS**

Further Pure Mathematics 2
TUESDAY 16 JANUARY 2007

4726/01

Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

2

1 It is given that $f(x) = \ln(3 + x)$.

(i) Find the exact values of $f(0)$ and $f'(0)$, and show that $f''(0) = -\frac{1}{9}$. [3]

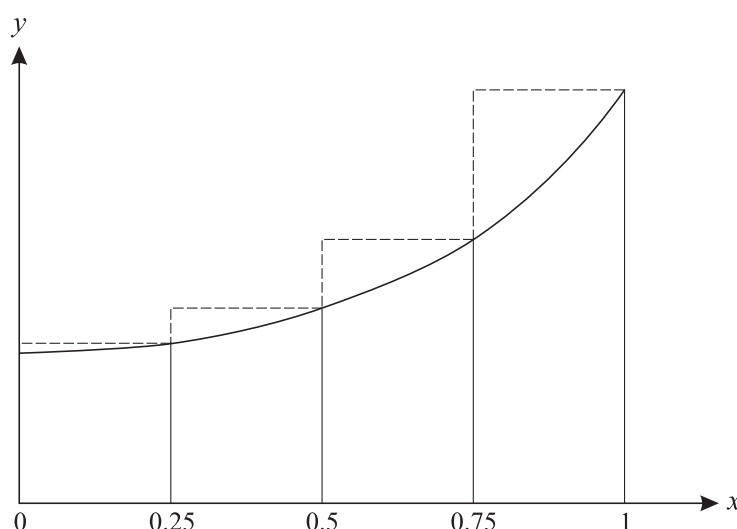
(ii) Hence write down the first three terms of the Maclaurin series for $f(x)$, given that $-3 < x \leq 3$. [2]

2 It is given that $f(x) = x^2 - \tan^{-1} x$.

(i) Show by calculation that the equation $f(x) = 0$ has a root in the interval $0.8 < x < 0.9$. [2]

(ii) Use the Newton-Raphson method, with a first approximation 0.8, to find the next approximation to this root. Give your answer correct to 3 decimal places. [4]

3



The diagram shows the curve with equation $y = e^{x^2}$, for $0 \leq x \leq 1$. The region under the curve between these limits is divided into four strips of equal width. The area of this region under the curve is A .

(i) By considering the set of rectangles indicated in the diagram, show that an upper bound for A is 1.71. [3]

(ii) By considering an appropriate set of four rectangles, find a lower bound for A . [3]

4 (i) On separate diagrams, sketch the graphs of $y = \sinh x$ and $y = \operatorname{cosech} x$. [3]

(ii) Show that $\operatorname{cosech} x = \frac{2e^x}{e^{2x} - 1}$, and hence, using the substitution $u = e^x$, find $\int \operatorname{cosech} x \, dx$. [6]

3

5 It is given that, for non-negative integers n ,

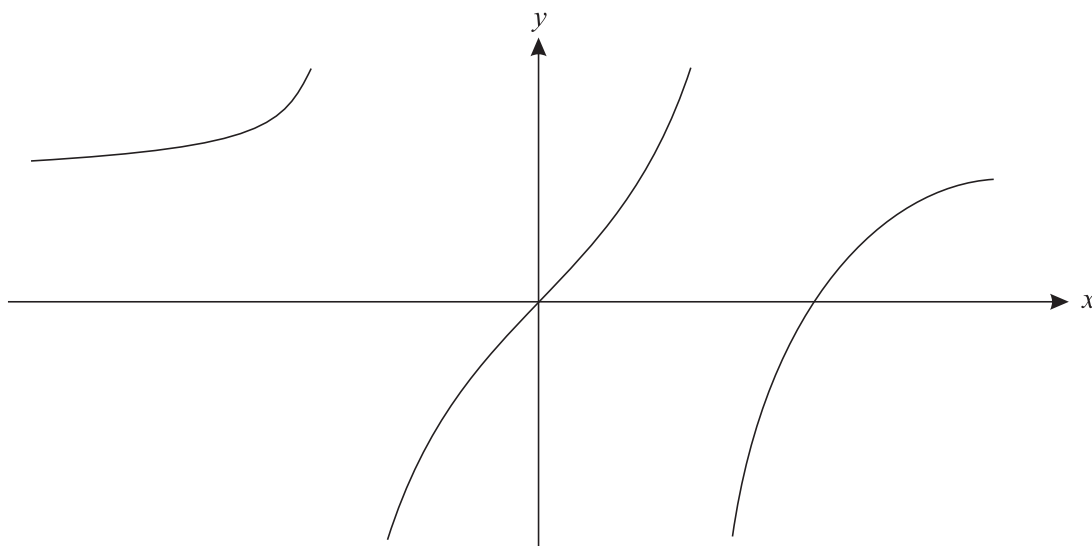
$$I_n = \int_0^{\frac{1}{2}\pi} x^n \cos x \, dx.$$

(i) Prove that, for $n \geq 2$,

$$I_n = \left(\frac{1}{2}\pi\right)^n - n(n-1)I_{n-2}. \quad [5]$$

(ii) Find I_4 in terms of π . [4]

6



The diagram shows the curve with equation $y = \frac{2x^2 - 3ax}{x^2 - a^2}$, where a is a positive constant.

(i) Find the equations of the asymptotes of the curve. [3]

(ii) Sketch the curve with equation

$$y^2 = \frac{2x^2 - 3ax}{x^2 - a^2}.$$

State the coordinates of any points where the curve crosses the axes, and give the equations of any asymptotes. [5]

7 (i) Express $\frac{1-t^2}{t^2(1+t^2)}$ in partial fractions. [4]

(ii) Use the substitution $t = \tan \frac{1}{2}x$ to show that

$$\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{\cos x}{1 - \cos x} \, dx = \sqrt{3} - 1 - \frac{1}{6}\pi. \quad [5]$$

4

- 8 (i) Define $\tanh y$ in terms of e^y and e^{-y} . [1]
- (ii) Given that $y = \tanh^{-1} x$, where $-1 < x < 1$, prove that $y = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$. [3]
- (iii) Find the exact solution of the equation $3 \cosh x = 4 \sinh x$, giving the answer in terms of a logarithm. [2]
- (iv) Solve the equation
- $$\tanh^{-1} x + \ln(1-x) = \ln\left(\frac{4}{5}\right). \quad [3]$$

- 9 The equation of a curve, in polar coordinates, is

$$r = \sec \theta + \tan \theta, \quad \text{for } 0 \leq \theta \leq \frac{1}{3}\pi.$$

- (i) Sketch the curve. [2]
- (ii) Find the exact area of the region bounded by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{3}\pi$. [6]
- (iii) Find a cartesian equation of the curve. [3]

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