

3. (i)

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**. (2)

The matrix **B** represents an enlargement, scale factor -2 , with centre the origin.

(b) Write down the matrix **B**. (1)

(ii)

$$\mathbf{M} = \begin{pmatrix} 3 & k \\ -2 & 3 \end{pmatrix}, \text{ where } k \text{ is a positive constant.}$$

Triangle *T* has an area of 16 square units.

Triangle *T* is transformed onto the triangle *T'* by the transformation represented by the matrix **M**.

Given that the area of the triangle *T'* is 224 square units, find the value of *k*. (3)



7. The parabola C has cartesian equation $y^2 = 4ax$, $a > 0$

The points $P(ap^2, 2ap)$ and $P'(ap^2, -2ap)$ lie on C .

(a) Show that an equation of the normal to C at the point P is

$$y + px = 2ap + ap^3 \tag{5}$$

(b) Write down an equation of the normal to C at the point P' . (1)

The normal to C at P meets the normal to C at P' at the point Q .

(c) Find, in terms of a and p , the coordinates of Q . (2)

Given that S is the focus of the parabola,

(d) find the area of the quadrilateral $SPQP'$. (3)



8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$, is a general point on H .

An equation for the tangent to H at P is given by

$$y = -\frac{1}{t^2}x + \frac{2c}{t}$$

The points A and B lie on H .

The tangent to H at A and the tangent to H at B meet at the point $\left(-\frac{6}{7}c, \frac{12}{7}c\right)$.

Find, in terms of c , the coordinates of A and the coordinates of B .

(5)



9. (a) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^n (r+1)2^{r-1} = n2^n$$

(5)

(b) A sequence of numbers is defined by

$$\begin{aligned} u_1 &= 0, & u_2 &= 32, \\ u_{n+2} &= 6u_{n+1} - 8u_n & n &\geq 1 \end{aligned}$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^{n+1} - 2^{n+3}$$

(7)



