

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS****Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education****MATHEMATICS****4737**

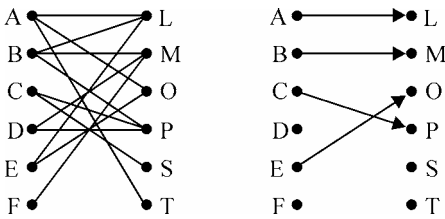
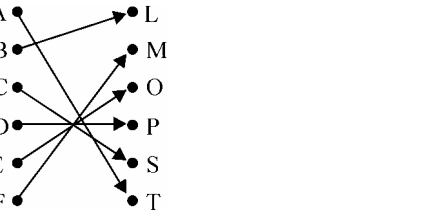
Decision Mathematics 2

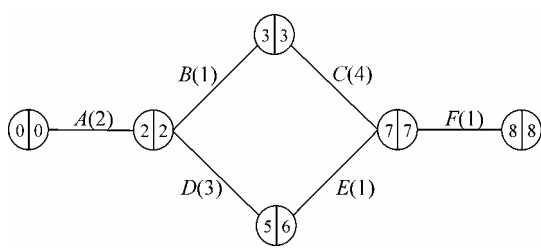
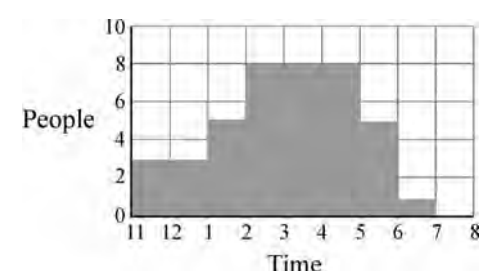
MARK SCHEME

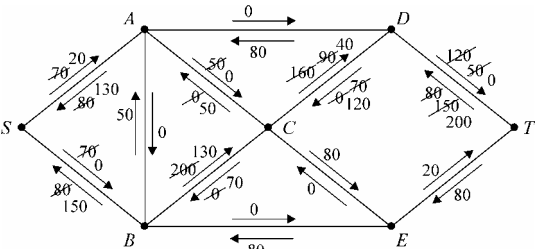
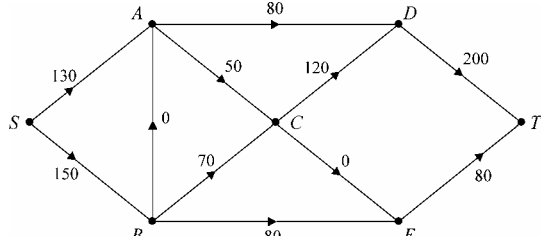
Specimen Paper

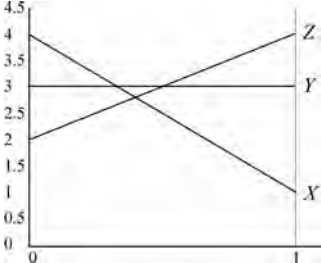
MAXIMUM MARK	72
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This mark scheme consists of 5 printed pages and 3 blank pages.

<p>1</p>	<p>(i)</p> 	<p>M1 A1 B1</p>	<p>For attempt at the bipartite graph For correct graph 3 For the correct incomplete matching</p>																																																																																																																																													
	<p>(ii) Alternating paths are: S-C=P-D and T-A=L-B=M-F or S-C=P-B=M-D and T-A=L-B=P-D=M-F or S-C=P-B=M-F and T-A=L-B=P-D or T-A=L-B=M-D and S-C=P-D=M-F or T-A=L-B=M-F and S-C=P-D</p> 	<p>M1 A1 A1 B1</p>	<p>For attempt at an alternating path For one correct path For the second path correct 4 For correct matching</p>																																																																																																																																													
	<p>(iii)</p> <table border="1" data-bbox="279 884 550 1086"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> </tr> </thead> <tbody> <tr> <th>L</th> <td>0</td> <td>0</td> <td>1</td> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <th>M</th> <td>1</td> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <th>O</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <th>P</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <th>S</th> <td>1</td> <td>1</td> <td>0</td> <td>1</td> <td>1</td> <td>1</td> </tr> <tr> <th>T</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table>		A	B	C	D	E	F	L	0	0	1	1	0	1	M	1	0	1	0	1	0	O	0	1	1	1	0	1	P	1	0	0	0	1	1	S	1	1	0	1	1	1	T	0	1	1	1	1	1	<p>M1 A1</p>	<p>For appropriate zeros and ones (e.g.) to correspond with minimum cost matching For a correct table 9</p>																																																																																												
	A	B	C	D	E	F																																																																																																																																										
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M	1	0	1	0	1	0																																																																																																																																										
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T	0	1	1	1	1	1																																																																																																																																										
<p>2</p>	<p>(i) Adding a dummy column gives</p> <table border="0" data-bbox="598 1120 774 1254"> <tr><td>4</td><td>3</td><td>2</td><td>4</td><td>6</td></tr> <tr><td>3</td><td>5</td><td>4</td><td>2</td><td>6</td></tr> <tr><td>3</td><td>6</td><td>3</td><td>3</td><td>6</td></tr> <tr><td>2</td><td>6</td><td>4</td><td>3</td><td>6</td></tr> <tr><td>2</td><td>5</td><td>3</td><td>4</td><td>6</td></tr> </table> <p>Reducing:</p> <table border="0" data-bbox="247 1265 774 1400"> <tr><td>2</td><td>1</td><td>0</td><td>2</td><td>4</td><td>2</td><td>0</td><td>0</td><td>2</td><td>1</td></tr> <tr><td>1</td><td>3</td><td>2</td><td>0</td><td>4</td><td>1</td><td>2</td><td>2</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>3</td><td>0</td><td>0</td><td>3</td><td>0</td><td>2</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>4</td><td>2</td><td>1</td><td>4</td><td>0</td><td>3</td><td>2</td><td>1</td><td>1</td></tr> <tr><td>0</td><td>3</td><td>1</td><td>2</td><td>4</td><td>0</td><td>2</td><td>1</td><td>2</td><td>1</td></tr> </table> <p>Four lines are needed to cover zeros</p> <table border="0" data-bbox="406 1478 774 1612"> <tr><td>3</td><td>0</td><td>0</td><td>3</td><td>1</td><td>3</td><td>0</td><td>0</td><td>2</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>2</td><td>2</td><td>2</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>2</td><td>0</td><td>1</td><td>0</td><td>1</td><td>2</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>2</td><td>1</td><td>1</td><td>0</td><td>0</td><td>2</td><td>1</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>0</td><td>2</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td></tr> </table> <p>Hence Adam-Glasgow and Betty-Swansea and either Clive-Manchester, Dave-London or Clive-Manchester, Eleanor-London or Dave-London, Eleanor-Manchester</p> <p>(ii) Without Betty, reduced matrix is</p> <table border="0" data-bbox="614 1769 774 1881"> <tr><td>2</td><td>0</td><td>0</td><td>2</td></tr> <tr><td>0</td><td>2</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>3</td><td>2</td><td>1</td></tr> <tr><td>0</td><td>2</td><td>1</td><td>2</td></tr> </table> <p>Hence either Adam-Glasgow, Clive-Swansea, Dave-London, Eleanor-Manchester or Adam-Glasgow, Clive-Manchester, Dave-Swansea, Eleanor-London</p>	4	3	2	4	6	3	5	4	2	6	3	6	3	3	6	2	6	4	3	6	2	5	3	4	6	2	1	0	2	4	2	0	0	2	1	1	3	2	0	4	1	2	2	0	1	0	3	0	0	3	0	2	0	0	0	0	4	2	1	4	0	3	2	1	1	0	3	1	2	4	0	2	1	2	1	3	0	0	3	1	3	0	0	2	1	1	1	1	0	0	2	2	2	0	1	1	2	0	1	0	1	2	0	0	0	0	2	1	1	0	0	2	1	0	0	0	1	0	2	0	0	1	0	1	0	2	0	0	2	0	2	0	0	0	3	2	1	0	2	1	2	<p>B1 M1 M1 M1 A1 A1 M1 A1</p>	<p>For a dummy column (equal entries, ≥ 6) For reducing rows For reducing columns For covering zeros in the reduced matrix For correct augmentation process For these two allocations correct 7 For any one of the correct possibilities For new reduced matrix 2 For either of the correct new possibilities 9</p>
4	3	2	4	6																																																																																																																																												
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0	2	1	2																																																																																																																																													

3	(i)	<table border="1"> <tr> <td rowspan="3">2</td> <td>0</td> <td>0</td> <td>5</td> <td>5</td> </tr> <tr> <td>1</td> <td>0</td> <td>7</td> <td>7</td> </tr> <tr> <td>2</td> <td>0</td> <td>9</td> <td>9</td> </tr> <tr> <td rowspan="6">1</td> <td rowspan="2">0</td> <td>0</td> <td>$\min(12, 5) = 5$</td> <td rowspan="2">7</td> </tr> <tr> <td>1</td> <td>$\min(7, 7) = 7$</td> </tr> <tr> <td rowspan="2">1</td> <td>1</td> <td>$\min(8, 7) = 7$</td> <td rowspan="2">9</td> </tr> <tr> <td>2</td> <td>$\min(10, 9) = 9$</td> </tr> <tr> <td rowspan="2">2</td> <td>0</td> <td>$\min(9, 5) = 5$</td> <td rowspan="2">9</td> </tr> <tr> <td>2</td> <td>$\min(14, 9) = 9$</td> </tr> </table> <p>Route is (0; 0)–(1; 2)–(2; 2)–(3; 0) Maximum number of crates is 8</p>	2	0	0	5	5	1	0	7	7	2	0	9	9	1	0	0	$\min(12, 5) = 5$	7	1	$\min(7, 7) = 7$	1	1	$\min(8, 7) = 7$	9	2	$\min(10, 9) = 9$	2	0	$\min(9, 5) = 5$	9	2	$\min(14, 9) = 9$	<p>M1 A1 M1 A1 A1</p>	<p>For dealing with route min column For at least 6 minima correct For dealing with maximin column For Stage 1 section of table all correct For completely correct table</p>																																						
2	0	0		5	5																																																																					
	1	0		7	7																																																																					
	2	0	9	9																																																																						
1	0	0	$\min(12, 5) = 5$	7																																																																						
		1	$\min(7, 7) = 7$																																																																							
	1	1	$\min(8, 7) = 7$	9																																																																						
		2	$\min(10, 9) = 9$																																																																							
	2	0	$\min(9, 5) = 5$	9																																																																						
		2	$\min(14, 9) = 9$																																																																							
		<p>Route is (0; 0)–(1; 2)–(2; 2)–(3; 0) Maximum number of crates is 8</p>	<p>B1 B1</p>	<p>7</p>																																																																						
	(ii)	<p>New maximin values are 15, 7, 9, 12, 9, 9, 9 Hence new route is (0; 0)–(1; 0)–(2; 0)–(3; 0) New maximum number of crates is 9</p>	<p>M1 A1 A1</p>	<p>For appropriate re-calculation For correct new route For correct number</p>																																																																						
			<p>3 10</p>																																																																							
4	(i)	 <p>Minimum completion time is 8 hours Critical activities are A, B, C, F Start telephoning at 11.00 am</p>	<p>B1 M1 M1 A1 A1 B1 B1✓</p>	<p>For correct arcs and activities (activity on arc network or equivalent with activity at node) For correct process for forward pass For correct process for reverse pass For all early and late times correct For correct minimum time stated For correct critical activities For stating the appropriate time of day</p>																																																																						
	(ii)	 <p>Maximum number of people needed is 8</p>	<p>M1 A1</p>	<p>For resource histogram with axes labelled For correct heights 3, 3, 5, 8, 8, 8, 5, 1</p>																																																																						
	(iii)	<table border="1"> <thead> <tr> <th>Time</th> <th>H</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <td>10–11</td> <td>A</td> <td>A</td> <td>A</td> <td>D</td> <td>D</td> <td>D</td> </tr> <tr> <td>11–12</td> <td>A</td> <td>A</td> <td>A</td> <td>D</td> <td>D</td> <td>D</td> </tr> <tr> <td>12– 1</td> <td></td> <td>B</td> <td>B</td> <td>D</td> <td>D</td> <td>D</td> </tr> <tr> <td>1– 2</td> <td></td> <td>C</td> <td>C</td> <td>C</td> <td>C</td> <td>C</td> </tr> <tr> <td>2– 3</td> <td></td> <td>C</td> <td>C</td> <td>C</td> <td>C</td> <td>C</td> </tr> <tr> <td>3– 4</td> <td></td> <td>C</td> <td>C</td> <td>C</td> <td>C</td> <td>C</td> </tr> <tr> <td>4– 5</td> <td></td> <td>C</td> <td>C</td> <td>C</td> <td>C</td> <td>C</td> </tr> <tr> <td>5– 6</td> <td>E</td> <td>E</td> <td>E</td> <td></td> <td></td> <td></td> </tr> <tr> <td>6– 7</td> <td>F</td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p>Start telephoning at 10.00 am</p>	Time	H	1	2	3	4	5	10–11	A	A	A	D	D	D	11–12	A	A	A	D	D	D	12– 1		B	B	D	D	D	1– 2		C	C	C	C	C	2– 3		C	C	C	C	C	3– 4		C	C	C	C	C	4– 5		C	C	C	C	C	5– 6	E	E	E				6– 7	F						<p>M1 A1</p>	<p>For substantially correct attempt For a correct schedule</p>
Time	H	1	2	3	4	5																																																																				
10–11	A	A	A	D	D	D																																																																				
11–12	A	A	A	D	D	D																																																																				
12– 1		B	B	D	D	D																																																																				
1– 2		C	C	C	C	C																																																																				
2– 3		C	C	C	C	C																																																																				
3– 4		C	C	C	C	C																																																																				
4– 5		C	C	C	C	C																																																																				
5– 6	E	E	E																																																																							
6– 7	F																																																																									
			<p>B1</p>	<p>3</p>																																																																						
			<p>13</p>																																																																							

<p>5 (i) Capacity is $150 + 0 + 50 + 80 = 280$ litres/sec</p>	<p>M1 A1</p>	<p>2 For correct use of zero from AB For correct value 280</p>
<p>(ii) Maximum flow is ≤ 280 So flow of $200 + 100 = 300$ is not possible</p>	<p>M1 A1</p>	<p>2 For relevant use of max flow/min cut For completely correct proof</p>
<p>(iii)</p> 	<p>M1 A1 A1</p>	<p>3 For correct method for excess and backflow For all initial excess capacities correct For all initial backflows correct</p>
<p>(iv) Augment by 70 along $SBCDT$ (e.g.) New excesses and backflows are as shown above Now augment by 50 along $SACDT$ (e.g.) Final excesses and backflows are as shown above</p>	<p>B1 M1 M1 A1</p>	<p>4 For identifying a correct augmentation For modifying excesses and backflows For continuing the process as far as possible For a completely correct solution</p>
<p>(v)</p> 	<p>B1\checkmark</p>	<p>For showing the augmented flow correctly</p>
<p>The value of the augmented flow is 280 litres/sec, and so is the maximum possible</p>	<p>M1 A1</p>	<p>3 For comparing with results from (i) or (ii) For a completely correct explanation (Or equivalent explanation based on the disconnectedness of S and T in (iii))</p>

6	<p>(i)</p> <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr> <td></td> <td><i>X</i></td> <td><i>Y</i></td> <td><i>Z</i></td> <td>row min</td> </tr> <tr> <td><i>A</i></td> <td>1</td> <td>3</td> <td>4</td> <td>1</td> </tr> <tr> <td><i>B</i></td> <td>4</td> <td>3</td> <td>2</td> <td>2</td> </tr> <tr> <td><i>C</i></td> <td>3</td> <td>2</td> <td>1</td> <td>1</td> </tr> <tr> <td>- col max</td> <td>-4</td> <td>-3</td> <td>-4</td> <td></td> </tr> </table> <div style="text-align: center; margin-top: 5px;"> <p>←</p> <p>↑</p> </div> <p>Play-safe for Rose is <i>B</i> Play-safe for computer is <i>Y</i> Not stable as $-3 + 2 \neq 0$</p>		<i>X</i>	<i>Y</i>	<i>Z</i>	row min	<i>A</i>	1	3	4	1	<i>B</i>	4	3	2	2	<i>C</i>	3	2	1	1	- col max	-4	-3	-4		<p>B1 B1 B1</p>	<p>For correct statement For correct statement For use of max row min and min col max</p>
	<i>X</i>	<i>Y</i>	<i>Z</i>	row min																								
<i>A</i>	1	3	4	1																								
<i>B</i>	4	3	2	2																								
<i>C</i>	3	2	1	1																								
- col max	-4	-3	-4																									
(ii)	<p>Row <i>C</i> is dominated by row <i>B</i></p> <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr> <td></td> <td><i>X</i></td> <td><i>Y</i></td> <td><i>Z</i></td> </tr> <tr> <td><i>A</i></td> <td>1</td> <td>3</td> <td>4</td> </tr> <tr> <td><i>B</i></td> <td>4</td> <td>3</td> <td>2</td> </tr> </table>		<i>X</i>	<i>Y</i>	<i>Z</i>	<i>A</i>	1	3	4	<i>B</i>	4	3	2	<p>B1 B1</p>	<p>For correct statement or explanation For new matrix</p>													
	<i>X</i>	<i>Y</i>	<i>Z</i>																									
<i>A</i>	1	3	4																									
<i>B</i>	4	3	2																									
(iii)	<p>Expected pay-off with <i>X</i> is $1 \times a + 4(1 - a) = 4 - 3a$ and with <i>Y</i> is $3a + 3(1 - a) = 3$ and with <i>Z</i> is $4a + 2(1 - a) = 2 + 2a$</p>  <p>Required value of <i>a</i> is 0.4 Score of 1 or 2: play <i>A</i>; score of 3, 4 or 5: play <i>B</i>; Throw die again if it shows a six</p>	<p>B1 B1 B1 B1√ B1√ B1√</p>	<p>For showing given answer correctly For correct value 3 For correct expression $2 + 2a$ For correct diagram For correct value For a correct decision rule</p>																									
(iv)	<table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr> <td></td> <td><i>X</i></td> <td><i>Y</i></td> <td><i>Z</i></td> </tr> <tr> <td><i>A</i></td> <td>-1</td> <td>-3</td> <td>-4</td> </tr> <tr> <td><i>B</i></td> <td>-4</td> <td>-3</td> <td>-2</td> </tr> </table> <p>Add 4 to each value</p>		<i>X</i>	<i>Y</i>	<i>Z</i>	<i>A</i>	-1	-3	-4	<i>B</i>	-4	-3	-2	<p>B1 B1</p>	<p>For correct pay-off matrix for the computer For a correct explanation</p>													
	<i>X</i>	<i>Y</i>	<i>Z</i>																									
<i>A</i>	-1	-3	-4																									
<i>B</i>	-4	-3	-2																									
(v)	<p>$z = 0.6$ $p = 1.2 \Rightarrow P = -2.8$ The computer should choose <i>X</i> with probability 0.4 and <i>Z</i> with probability 0.6 On average the computer will lose no more than 2.8 points per game</p>	<p>B1 B1 B1 B1</p>	<p>For the correct value of <i>z</i> For the correct values of both <i>p</i> and <i>P</i> For a correct description of the strategy For a correct interpretation of <i>P</i></p>																									