



ADVANCED GCE
MATHEMATICS

4726/01

Further Pure Mathematics 2

WEDNESDAY 9 JANUARY 2008

Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of 4 printed pages.

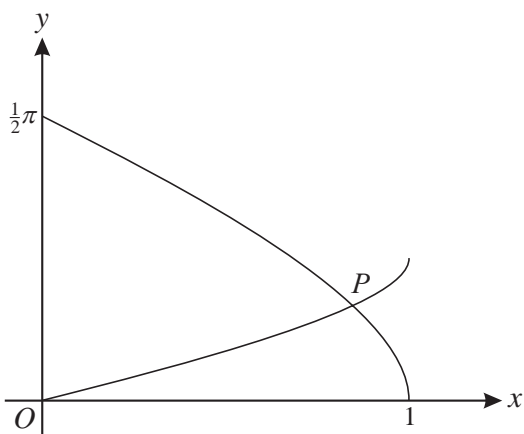
2

1 It is given that $f(x) = \ln(1 + \cos x)$.

(i) Find the exact values of $f(0)$, $f'(0)$ and $f''(0)$. [4]

(ii) Hence find the first two non-zero terms of the Maclaurin series for $f(x)$. [2]

2

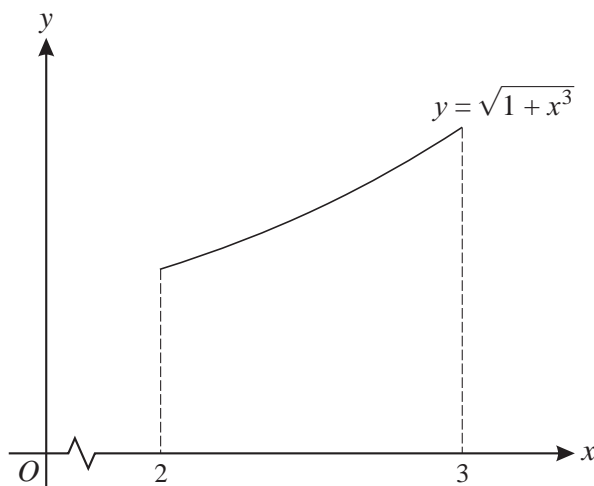


The diagram shows parts of the curves with equations $y = \cos^{-1} x$ and $y = \frac{1}{2} \sin^{-1} x$, and their point of intersection P .

(i) Verify that the coordinates of P are $(\frac{1}{2}\sqrt{3}, \frac{1}{6}\pi)$. [2]

(ii) Find the gradient of each curve at P . [3]

3



The diagram shows the curve with equation $y = \sqrt{1 + x^3}$, for $2 \leq x \leq 3$. The region under the curve between these limits has area A .

(i) Explain why $3 < A < \sqrt{28}$. [2]

(ii) The region is divided into 5 strips, each of width 0.2. By using suitable rectangles, find improved lower and upper bounds between which A lies. Give your answers correct to 3 significant figures. [4]

3

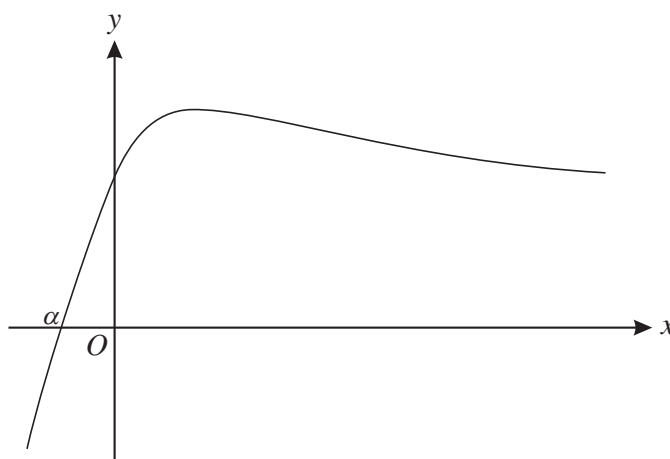
4 The equation of a curve, in polar coordinates, is

$$r = 1 + 2 \sec \theta, \quad \text{for } -\frac{1}{2}\pi < \theta < \frac{1}{2}\pi.$$

(i) Find the exact area of the region bounded by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{6}\pi$. [5]
[The result $\int \sec \theta \, d\theta = \ln |\sec \theta + \tan \theta|$ may be assumed.]

(ii) Show that a cartesian equation of the curve is $(x - 2)\sqrt{x^2 + y^2} = x$. [3]

5



The diagram shows the curve with equation $y = xe^{-x} + 1$. The curve crosses the x -axis at $x = \alpha$.

(i) Use differentiation to show that the x -coordinate of the stationary point is 1. [2]

α is to be found using the Newton-Raphson method, with $f(x) = xe^{-x} + 1$.

(ii) Explain why this method will not converge to α if an initial approximation x_1 is chosen such that $x_1 > 1$. [2]

(iii) Use this method, with a first approximation $x_1 = 0$, to find the next three approximations x_2 , x_3 and x_4 . Find α , correct to 3 decimal places. [5]

6 The equation of a curve is $y = \frac{2x^2 - 11x - 6}{x - 1}$.

(i) Find the equations of the asymptotes of the curve. [3]

(ii) Show that y takes all real values. [5]

4

7 It is given that, for integers $n \geq 1$,

$$I_n = \int_0^1 \frac{1}{(1+x^2)^n} dx.$$

(i) Use integration by parts to show that $I_n = 2^{-n} + 2n \int_0^1 \frac{x^2}{(1+x^2)^{n+1}} dx$. [3]

(ii) Show that $2nI_{n+1} = 2^{-n} + (2n-1)I_n$. [3]

(iii) Find I_2 in terms of π . [3]

8 (i) By using the definition of $\sinh x$ in terms of e^x and e^{-x} , show that

$$\sinh^3 x = \frac{1}{4} \sinh 3x - \frac{3}{4} \sinh x. \quad [4]$$

(ii) Find the range of values of the constant k for which the equation

$$\sinh 3x = k \sinh x$$

has real solutions other than $x = 0$. [3]

(iii) Given that $k = 4$, solve the equation in part (ii), giving the non-zero answers in logarithmic form. [3]

9 (i) Prove that $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$. [3]

(ii) Hence, or otherwise, find $\int \frac{1}{\sqrt{4x^2-1}} dx$. [2]

(iii) By means of a suitable substitution, find $\int \sqrt{4x^2-1} dx$. [6]

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (OCR) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

OCR is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.