



General Certificate of Education
Advanced Subsidiary Examination
June 2012

Mathematics

MPC2

Unit Pure Core 2

Wednesday 16 May 2012 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

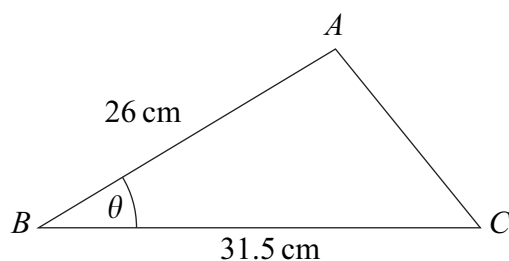
1 The arithmetic series

$$23 + 32 + 41 + 50 + \dots + 2534$$

has 280 terms.

- (a) Write down the common difference of the series. (1 mark)
- (b) Find the 100th term of the series. (2 marks)
- (c) Find the sum of the 280 terms of the series. (2 marks)
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2 The triangle ABC , shown in the diagram, is such that $AB = 26$ cm and $BC = 31.5$ cm.



The acute angle ABC is θ , where $\sin \theta = \frac{5}{13}$.

- (a) Calculate the area of triangle ABC . (2 marks)
- (b) Find the exact value of $\cos \theta$. (1 mark)
- (c) Calculate the length of AC . (3 marks)
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3 (a) Expand $\left(x^{\frac{3}{2}} - 1\right)^2$. (2 marks)

(b) Hence find $\int \left(x^{\frac{3}{2}} - 1\right)^2 dx$. (3 marks)

(c) Hence find the value of $\int_1^4 \left(x^{\frac{3}{2}} - 1\right)^2 dx$. (2 marks)



4 The n th term of a geometric series is u_n , where $u_n = 48\left(\frac{1}{4}\right)^n$.

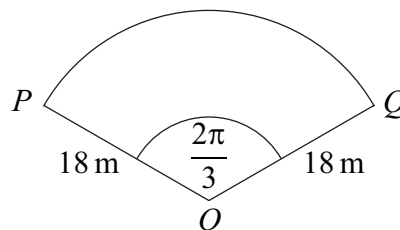
(a) Find the value of u_1 and the value of u_2 . (2 marks)

(b) Find the value of the common ratio of the series. (1 mark)

(c) Find the sum to infinity of the series. (2 marks)

(d) Find the value of $\sum_{n=4}^{\infty} u_n$. (3 marks)

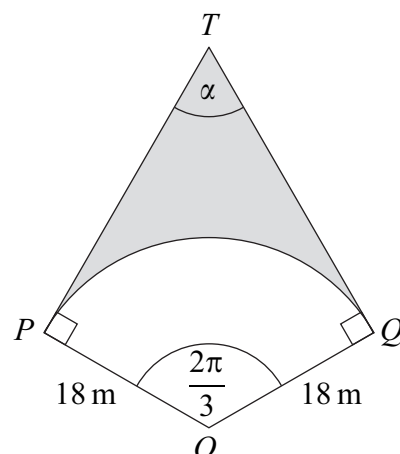
5 The diagram shows a sector OPQ of a circle with centre O .



The radius of the circle is 18 m and the angle POQ is $\frac{2\pi}{3}$ radians.

(a) Find the length of the arc PQ , giving your answer as a multiple of π . (2 marks)

(b) The tangents to the circle at the points P and Q meet at the point T , and the angles TPO and TQO are both right angles, as shown in the diagram below.



(i) Angle $PTQ = \alpha$ radians. Find α in terms of π . (1 mark)

(ii) Find the area of the shaded region bounded by the arc PQ and the tangents TP and TQ , giving your answer to three significant figures. (6 marks)

Turn over ►



- 6 At the point (x, y) , where $x > 0$, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^2 - \frac{4}{x^2} - 11$$

The point $P(2, 1)$ lies on the curve.

- (a) (i) Verify that $\frac{dy}{dx} = 0$ when $x = 2$. (1 mark)
- (ii) Find the value of $\frac{d^2y}{dx^2}$ when $x = 2$. (4 marks)
- (iii) Hence state whether P is a maximum point or a minimum point, giving a reason for your answer. (1 mark)
- (b) Find the equation of the curve. (4 marks)
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- 7 It is given that $(\tan \theta + 1)(\sin^2 \theta - 3 \cos^2 \theta) = 0$.

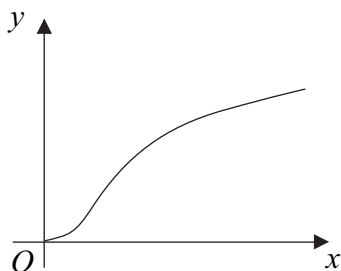
- (a) Find the possible values of $\tan \theta$. (4 marks)
- (b) Hence solve the equation $(\tan \theta + 1)(\sin^2 \theta - 3 \cos^2 \theta) = 0$, giving all solutions for θ , in degrees, in the interval $0^\circ \leq \theta \leq 180^\circ$. (3 marks)
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- 8 (a) Sketch the curve with equation $y = 7^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. (2 marks)

- (b) The curve C_1 has equation $y = 7^x$.
The curve C_2 has equation $y = 7^{2x} - 12$.
- (i) By forming and solving a quadratic equation, prove that the curves C_1 and C_2 intersect at exactly one point. State the y -coordinate of this point. (4 marks)
- (ii) Use logarithms to find the x -coordinate of the point of intersection of C_1 and C_2 , giving your answer to three significant figures. (2 marks)



- 9 The diagram shows part of a curve whose equation is $y = \log_{10}(x^2 + 1)$.



- (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^1 \log_{10}(x^2 + 1) \, dx$$

giving your answer to three significant figures. (4 marks)

- (b) The graph of $y = 2 \log_{10} x$ can be transformed into the graph of $y = 1 + 2 \log_{10} x$ by means of a translation. Write down the vector of the translation. (1 mark)

- (c) (i) Show that $\log_{10}(10x^2) = 1 + 2 \log_{10} x$. (2 marks)

- (ii) Show that the graph of $y = 2 \log_{10} x$ can also be transformed into the graph of $y = 1 + 2 \log_{10} x$ by means of a **stretch**, and describe the stretch. (4 marks)

- (iii) The curve with equation $y = 1 + 2 \log_{10} x$ intersects the curve $y = \log_{10}(x^2 + 1)$ at the point P . Given that the x -coordinate of P is positive, find the gradient of the line OP , where O is the origin. Give your answer in the form $\log_{10}\left(\frac{a}{b}\right)$, where a and b are integers. (4 marks)

