OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4730**: Mechanics 3

Advanced GCE

Mark Scheme for June 2015

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Answer			Marks	Guidance	
1	**(i)**	impulse momentum diagram	M1 A1	right-angled triangle with angle α and sides labelled 3, v and I/0.2 or 0.6, 0.2v and I	correct orientation, α and one side labelled correctly, right angle implied first two marks may be implied by correct working
		tan α = I/(0.2x3) I = 0.25 shown	M1 A1 **[4]**	**AG**	
		OR $0.2 \times 3 = 0.2v\cos\alpha$ and $I = 0.2v\sin\alpha$ $\frac{I}{0.2 \times 3} = \tan\alpha$ I = 0.25	M1 A1 M1 A1	resolve parallel or perp to dir of motion both attempt to manipulate **AG**	
	(ii)	cosα = 3/v (speed) = 3.25 m s^{-1}	M1 A1 **[2]**	or using Pythagoras, with 3 and 1.25 oe	
2	**m**	Moments about B for BC $75L\cos\beta = 50x2L\sin\beta$ $\tan\beta = 3/4$	M1 A1 A1 **[3]**	2 terms involving sinβ and cosβ, 75 and 50 WWW **AG**	allow sin/cos error/ sign error allow missing L
	(ii)	moments about A for both rods $WL\cos\alpha + 75(2L\cos\alpha + L\cos\beta) =$ $50(2L\sin\alpha + 2L\sin\beta)$ correct values for sin/cos α/β attempt to solve (W=) 90 (N) OR 'X' = 50 N to right on AB oe 'Y' = 75 N down on AB oe Moments about A for AB $WL\cos\alpha + 75 \times 2L\cos\alpha = 50 \times 2L\sin\alpha$ (W=) 90 (N)	M1* A1 B1 *M1 A1 **[5]** B1 B1 M1 A1 A1	all (5) terms present; each term involves sin/cos α/β. Dim correct: no extra terms dep M1A1 dep B1 also sc B1 for magnitudes if directions wrong/missing involves W, 75, 50, sin α and cos α. dimensionally correct; no extra terms with substitution for α	allow sin/cos, L/2L, sign errors L may be cancelled all 4 seen all values substituted 50 & 75 may be seen on diagram in (i) L may be cancelled

Answer			Marks	Guidance	
3	(i)	use of $T = \frac{\lambda x}{l}$	M1	used at least once	
		$T = \frac{10\times0.2}{0.4} + \frac{12\times0.1}{0.5}$	A1		
		W = 7.4 N	A1	CAO **AG**	
		use of $E = \frac{\lambda x^2}{2l}$	M1	used at least once	
		$E = \frac{10(0.2)^2}{2\times0.4} + \frac{12(0.1)^2}{2\times0.5}$	A1		may see 0.5 + 0.12
		E = 0.62 (J)	A1 **[6]**		
	(ii)	use of $F = ma$ when further extension is x	M1*	allow sign errors, 'm' wrong	OR, when total length of string is x
		$7.4 - \frac{10\times(x+0.2)}{0.4} - \frac{12\times(x+0.1)}{0.5} = \frac{7.4}{g}a$	A1	'F' correct	$7.4 - \frac{10\times(x-0.4)}{0.4} - \frac{12\times(x-0.5)}{0.5} = \frac{7.4}{g}a$
		$a = -\frac{49g}{7.4}x$	A1	accept a = − 64.89...x, $a = -\frac{2401}{37}x$ oe	$a = -\frac{49g}{7.4}(x-0.6)$
		SHM: $\omega^2 = \frac{49g}{7.4}$ (or $\frac{2401}{37}$ or 64.89189)	A1	dep on all first 3 marks	SHM about x = 0.6, and ω^2 given
		Use of $T = \frac{2\pi}{\omega}$	*M1	must subst for their ω	
		period is 0.780 (secs) $\frac{2\pi\sqrt{37}}{49}$	A1	allow if ω correct	0.77998
		all subsequent motion is SHM because string does not become slack	B1 **[7]**	justified at some point	
4	(i)	$-\frac{v}{8} = 0.4\frac{dv}{dt}$	M1*	allow sign error, allow 0.4a	
			A1		
		$t = -3.2\int\frac{1}{v}dv$	*M1	attempt to separate variables and integrate	or $t = -3.2\int_{10}^{5}\frac{1}{v}dv$
		$t = -3.2\ln v + 3.2\ln 10$	A1		$t = -3.2\ln v$; limits used correctly
		time taken = 3.2ln2 or 2.22 (s)	A1 **[5]**		2.21807...

Answer			Marks	Guidance	
	(ii)	$-\frac{v}{8} = 0.4v\frac{dv}{dx}$	M1*	allow sign error	
			A1		
		$x = -3.2\int dv$	*M1	attempt to separate variables and integrate	
		$x = -3.2$ v + 32	A1	x = 16 when v = 5.	
		ave speed = x/(i)	*M1		their x evaluated
		ave speed = 7.21	A1 **[6]**		accept 5/ln2
		OR			
		$\frac{dx}{dt} = 10e^{-\frac{t}{3.2}}$	M1*	for M1, ft from (i), must contain ln term	
			A1		
		$x = 10\int e^{-\frac{t}{3.2}}\,dt$	*M1	attempt to separate variables and integrate	
		$x = 32\left(1 - e^{-\frac{t}{3.2}}\right)$	A1	must show constant or use limits correctly	x = 16 when t = 3.2 ln(2)
		ave speed = x/(i)	*M1		
		ave speed = 7.21	A1	dep all 5 previous marks	accept 5/ln2
5	**(i)**	use of conservation of momentum	M1*	must be 3 non-zero terms	allow sign errors, m/2m errors, sin/cos
		$2ma\cos\alpha - mb\cos\beta = m$x2xcos45°	A1		
		use of NEL	M1*	must be 3 non-zero terms, and 'e' in correct postion	allow sign errors, sin/cos,
		2cos 45° − 0 = − 2/3 (-$b\cos\beta - a\cos\alpha$)	A1		
		attempt to eliminate $a\cos\alpha$ or $b\cos\beta$	*M1	dep both previous M1 marks	
		$a\cos\alpha = 5\sqrt{2}/6$	A1	**AG**	
		$b\cos\beta = 2\sqrt{2}/3$ oe	A1 **[7]**	dep final M1 and www	
	(ii)	$a\sin\alpha = 2$	B1		
		attempt to solve $a\sin\alpha = 2$ and $a\cos\alpha = 5\sqrt{2}/6$	M1	need to eliminate a or α	
		$a = 2.32$	A1		2.321398...,
		$\alpha = 59.5°$	A1 **[4]**	accept 1.03 radians	59.49104...°, 1.0383...rad

Answer			Marks	Guidance	
6	**(i)**	$a = 0.6$ (m) $\omega = 4$ max vel = $a\omega = 2.4$ (m s^{-1})	B1 B1 B1ft **[3]**	 accept sight of $\frac{\pi}{0.25\pi}$ or $\frac{2\pi}{0.5\pi}$ ft from wrong a and/or ω	
	(ii)	*must use their a and ω from (i) unless defined differently in (ii)* $x = 0.6\cos 4x0.7$ $x = -0.565$ $v = -0.6x4x\sin 4x0.7$ $v = -0.804$	 M1 A1 M1 A1 **[4]**	 use of $a\cos\omega t$; complete method use of $(-)\ a\omega\sin\omega t$ or $v^2 = \omega^2(a^2 - x^2)$ if v^2 formula used, direction of v needs to be made clear.	 or $a\sin(\omega t + \varepsilon)$, with $\varepsilon = \pm \pi/2$ −0.565333... or $(-)a\omega\cos(\omega t + \varepsilon)$, with $\varepsilon = \pm \pi/2$; allow M1ft from wrong formula for x −0.80397.....
	(iii)	*do not accept answers from wrong working* t and x for one point t and x for second point correctly giving precisely 2 other occasions, with x and t values matching sc, if < 3 scored, both t values B2 or one t value B1 or $x = 0.565$ B1 of B0 scored allow B1 for number of other occasions shown to be 2	 B2 B1 B1 **[4]**	 values of t are = 0.0854, 0.871 values of x are 0.565, −0.565 dep first 3 marks ignore wrong values “ “	 $\pi/4$ –0 7, $\pi/2$ - 0.7 ignore ref to point when $t = 0.7$ can show on diagram P has this speed 4 times in 1 period (1.570 s) so 2 other times in $0 < t < 1$

Answer			Marks	Guidance	
7	(i)	using $F = ma$	M1	must have the right 3 terms; allow sign error / sin for cos for M1	
		$T - 0.2g\cos\theta = 0.2v^2/0.5$	A1		
		by energy	M1		
		$\frac{1}{2}\times 0.2u^2 = \frac{1}{2}\times 0.2v^2 + 0.2g \times 0.5(1 - \cos\theta)$	A1		$v^2 = u^2 - 9.8(1 - \cos\theta)$
		$T = 5.88\cos\theta + 0.4u^2 - 3.92$	A1 **[5]**	**AG** with no errors and no gaps in argument	
	(ii)	when $\theta = 180°$, $5.88\cos\theta + 0.4u^2 - 3.92 = 0$	M1	allow inequalities for M1A1	
		$-5.88 + 0.4u^2 - 3.92 = 0$	A1		
		min u is 4.95 (m s^{-1})	A1 **[3]**	$\frac{7}{2}\sqrt{2}$	4.9497... Not > 4.95
		OR, at top, $mg = \frac{mv^2}{r}$, so $v^2 = 0.5g$	B1	allow inequalities for B1M1	
		by energy $\frac{1}{2}0.2u^2 = \frac{1}{2}0.2 \times 0.5g + 0.2g$	M1		
		min u is 4.95 (m s^{-1})	A1		
	(iii)	$5.88\cos\theta + 0.4\times 12.25 - 3.92 = 0$	M1		
		$\cos\theta = (3.92 - 4.9)/5.88$ $(= -1/6)$	A1	might see θ = 99.6° or 1.74 radians	99.49406...°, 1.73824...rads
		use energy eq^n from **(i)**	M1	accept use of their θ	
		$\frac{1}{2}\times 0.2\times 3.5^2 = \frac{1}{2}\times 0.2v^2 + 0.2g \times 0.5(1 - \cos\theta)$			
		$v = 0.904$ m s^{-1}	A1 **[4]**	$\frac{7}{30}\sqrt{15}$	0.903696...
		OR use T equation from (i)	M1		
		$0 - 0.2g(-1/6) = 0.2v^2/0.5$			
		$v = 0.904$ m s^{-1}	A1		

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

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