OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4726

Further Pure Mathematics 2

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Monday 10 January 2011
Morning

Duration: 1 hour 30 minutes

* 4 7 2 6 *

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Use the substitution $t = \tan\frac{1}{2}x$ to find $\displaystyle\int \frac{1}{1+\sin x+\cos x}\,\mathrm{d}x$. **[5]**

2 It is given that $\mathrm{f}(x) = \tanh^{-1} x$.

(i) Show that $\mathrm{f}'''(x) = \dfrac{2\left(1+3x^2\right)}{\left(1-x^2\right)^3}$. **[5]**

(ii) Hence find the Maclaurin series for $\mathrm{f}(x)$, up to and including the term in x^3. **[3]**

3 The function f is defined by $\mathrm{f}(x) = \dfrac{5ax}{x^2+a^2}$, for $x \in \mathbb{R}$ and $a > 0$.

(i) For the curve with equation $y = \mathrm{f}(x)$,

(a) write down the equation of the asymptote, **[1]**

(b) find the range of values that y can take. **[4]**

(ii) For the curve with equation $y^2 = \mathrm{f}(x)$, write down

(a) the equation of the line of symmetry, **[1]**

(b) the maximum and minimum values of y, **[2]**

(c) the set of values of x for which the curve is defined. **[1]**

4 **(i)** Use the definitions of hyperbolic functions in terms of exponentials to prove that

$$8\sinh^4 x \equiv \cosh 4x - 4\cosh 2x + 3.$$ **[4]**

(ii) Solve the equation

$$\cosh 4x - 3\cosh 2x + 1 = 0,$$

giving your answer(s) in logarithmic form. **[5]**

5 The equation

$$x^3 - 5x + 3 = 0 \qquad \text{(A)}$$

may be solved by the Newton-Raphson method. Successive approximations to a root are denoted by $x_1, x_2, \ldots, x_n, \ldots$.

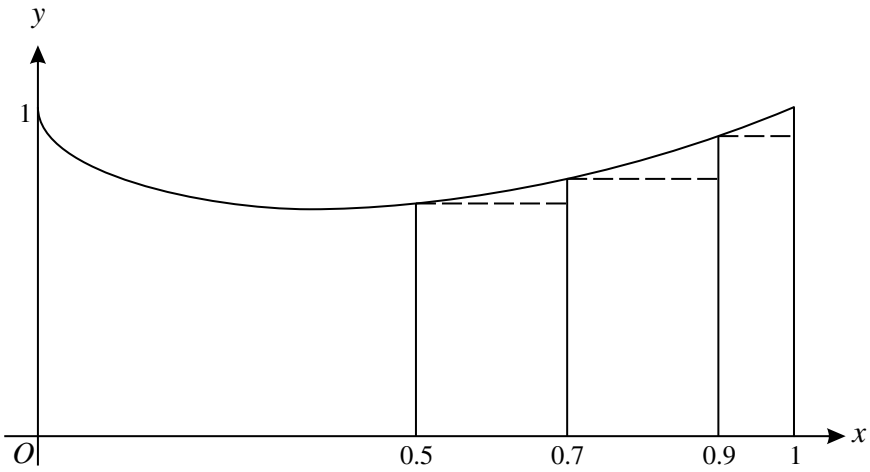
(i) Show that the Newton-Raphson formula can be written in the form $x_{n+1} = \mathrm{F}(x_n)$, where

$$\mathrm{F}(x) = \frac{2x^3-3}{3x^2-5}.$$ **[3]**

(ii) Find $\mathrm{F}'(x)$ and hence verify that $\mathrm{F}'(\alpha) = 0$, where α is any one of the roots of equation (A). **[3]**

(iii) Use the Newton-Raphson method to find the root of equation (A) which is close to 2. Write down sufficient approximations to find the root correct to 4 decimal places. **[3]**

6



The diagram shows the curve $y = \mathrm{f}(x)$, defined by

$$\mathrm{f}(x) = \begin{cases} x^x & \text{for } 0 < x \leqslant 1, \\ 1 & \text{for } x = 0. \end{cases}$$

(i) By first taking logarithms, show that the curve has a stationary point at $x = \mathrm{e}^{-1}$. **[3]**

The area under the curve from $x = 0.5$ to $x = 1$ is denoted by A.

(ii) By considering the set of three rectangles shown in the diagram, show that a lower bound for A is 0.388. **[2]**

(iii) By considering another set of three rectangles, find an upper bound for A, giving 3 decimal places in your answer. **[2]**

The area under the curve from $x = 0$ to $x = 0.5$ is denoted by B.

(iv) Draw a diagram to show rectangles which could be used to find lower and upper bounds for B, using not more than three rectangles for each bound. (You are not required to find the bounds.) **[3]**

7 A curve has polar equation $r = 1 + \cos 3\theta$, for $-\pi < \theta \leqslant \pi$.

(i) Show that the line $\theta = 0$ is a line of symmetry. **[2]**

(ii) Find the equations of the tangents at the pole. **[3]**

(iii) Find the exact value of the area of the region enclosed by the curve between $\theta = -\frac{1}{3}\pi$ and $\theta = \frac{1}{3}\pi$. **[5]**

8 **(i)** Without using a calculator, show that $\sinh(\cosh^{-1} 2) = \sqrt{3}$. **[2]**

(ii) It is given that, for non-negative integers n,

$$I_n = \int_0^{\beta} \cosh^n x \, \mathrm{d}x, \quad \text{where } \beta = \cosh^{-1} 2.$$

Show that $nI_n = 2^{n-1}\sqrt{3} + (n-1)I_{n-2}$, for $n \geqslant 2$. **[6]**

(iii) Evaluate I_5, giving your answer in the form $k\sqrt{3}$. **[4]**

There are no questions printed on this page.