Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

General Certificate of Education
Advanced Level Examination
January 2011

Mathematics MD02

Unit Decision 2

Wednesday 26 January 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Answer **all** questions in the spaces provided.

1 A group of workers is involved in a decorating project. The table shows the activities involved. Each worker can perform any of the given activities.

Activity	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
Duration (days)	2	5	6	7	9	4	3	2	3	2	3	1
Number of workers required	6	3	5	2	5	2	4	4	5	3	2	4

The activity network for the project is given in **Figure 1** below.

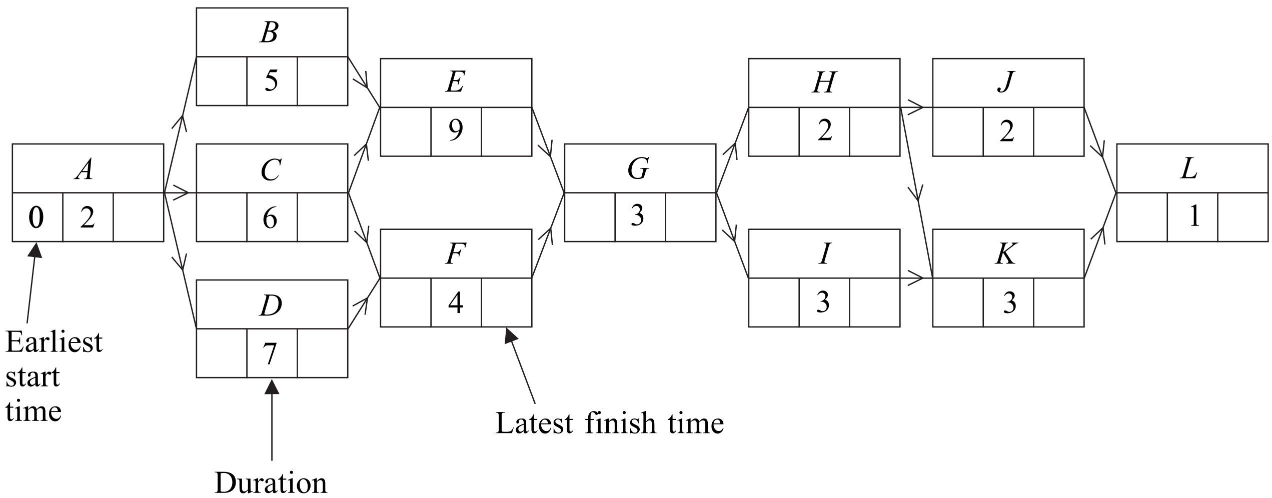
(a) Find the earliest start time and the latest finish time for each activity, inserting their values on **Figure 1**. *(4 marks)*

(b) Hence find:

(i) the critical path;

(ii) the float time for activity *D*. *(3 marks)*

(a) **Figure 1**



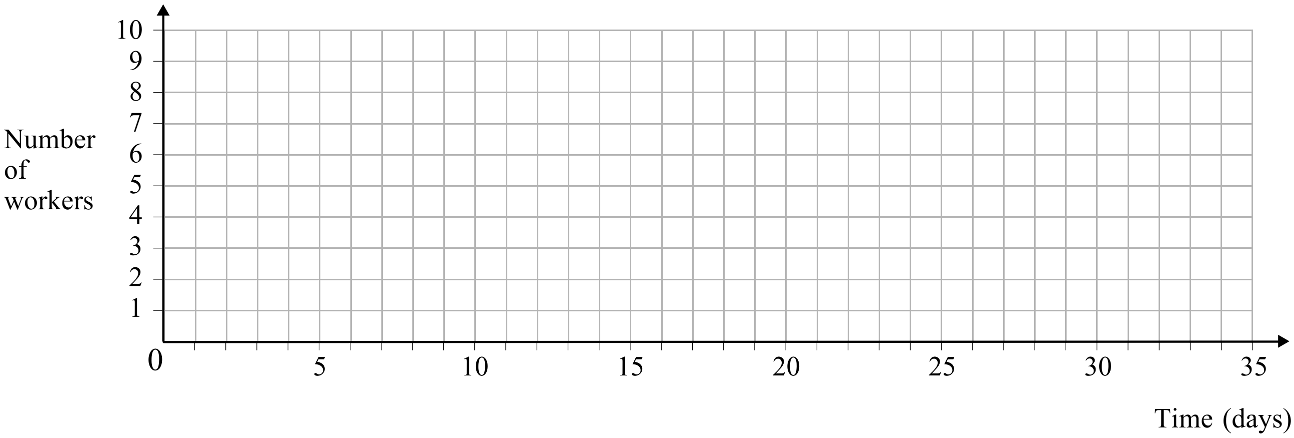
(b) (i) The critical path is ....

(ii) The float time for activity *D* is ....

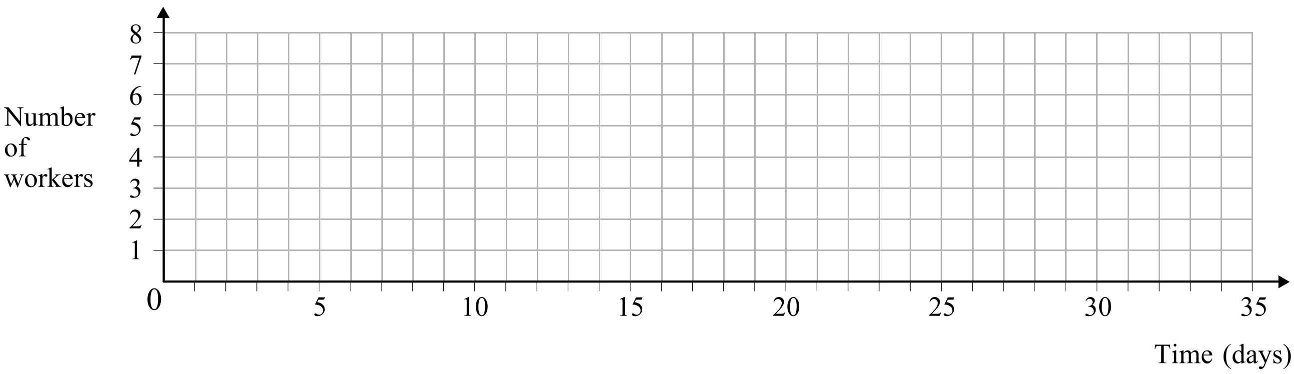
(c) Given that each activity starts as early as possible and assuming that there is no limit to the number of workers available, draw a resource histogram for the project on **Figure 2** below, indicating clearly which activities are taking place at any given time. *(4 marks)*

(d) It is later discovered that there are only 8 workers available at any time. Use resource levelling to construct a new resource histogram on **Figure 3** below, showing how the project can be completed with the minimum extra time. State the minimum extra time required. *(3 marks)*

(c) **Figure 2**



(d) **Figure 3**



The minimum extra time required is ....

2 A farmer has five fields. He intends to grow a different crop in each of four fields and to leave one of the fields unused. The farmer tests the soil in each field and calculates a score for growing each of the four crops. The scores are given in the table below.

	Field A	Field B	Field C	Field D	Field E
Crop 1	16	12	8	18	14
Crop 2	20	15	8	16	12
Crop 3	9	10	12	17	12
Crop 4	18	11	17	15	19

The farmer's aim is to maximise the total score for the four crops.

(a) (i) Modify the table of values by first subtracting each value in the table above from 20 and then adding an extra row of equal values. *(1 mark)*

(ii) Explain why the Hungarian algorithm can now be applied to the new table of values to maximise the total score for the four crops. *(3 marks)*

(b) (i) By reducing **rows** first, show that the table from part **(a)(i)** becomes

2	6	10	0	p
0	5	12	4	8
8	7	5	0	q
1	8	2	4	0
0	0	0	0	0

State the values of the constants p and q. *(2 marks)*

(ii) Show that the zeros in the table from part **(b)(i)** can be covered by one horizontal and three vertical lines, and use the Hungarian algorithm to decide how the four crops should be allocated to the fields. *(6 marks)*

(iii) Hence find the maximum possible total score for the four crops. *(1 mark)*

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3 Two people, Rhona and Colleen, play a zero-sum game. The game is represented by the following pay-off matrix for Rhona.

	Colleen			
	Strategy	**C_1**	**C_2**	**C_3**
	R_1	2	6	4
Rhona	**R_2**	3	-3	-1
	R_3	x	$x+3$	3

It is given that $x < 2$.

(a) (i) Write down the three row minima. *(1 mark)*

(ii) Show that there is no stable solution. *(3 marks)*

(b) Explain why Rhona should never play strategy R_3. *(1 mark)*

(c) (i) Find the optimal mixed strategy for Rhona. *(7 marks)*

(ii) Find the value of the game. *(1 mark)*

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4 The Simplex method is to be used to maximise $P = 3x + 2y + z$ subject to the constraints

$$-x + y + z \leqslant 4$$

$$2x + y + 4z \leqslant 10$$

$$4x + 2y + 3z \leqslant 21$$

The initial Simplex tableau is given below.

P	***x***	***y***	***z***	***s***	***t***	***u***	***value***
1	−3	−2	−1	0	0	0	0
0	−1	1	1	1	0	0	4
0	2	1	4	0	1	0	10
0	4	2	3	0	0	1	21

(a) (i) The first pivot is to be chosen from the x-column. Identify the pivot and explain why this particular value is chosen. *(2 marks)*

(ii) Perform one iteration of the Simplex method and explain how you know that the optimal value has not been reached. *(5 marks)*

(b) (i) Perform one further iteration. *(4 marks)*

(ii) Interpret the final tableau and write down the initial inequality that still has slack. *(4 marks)*

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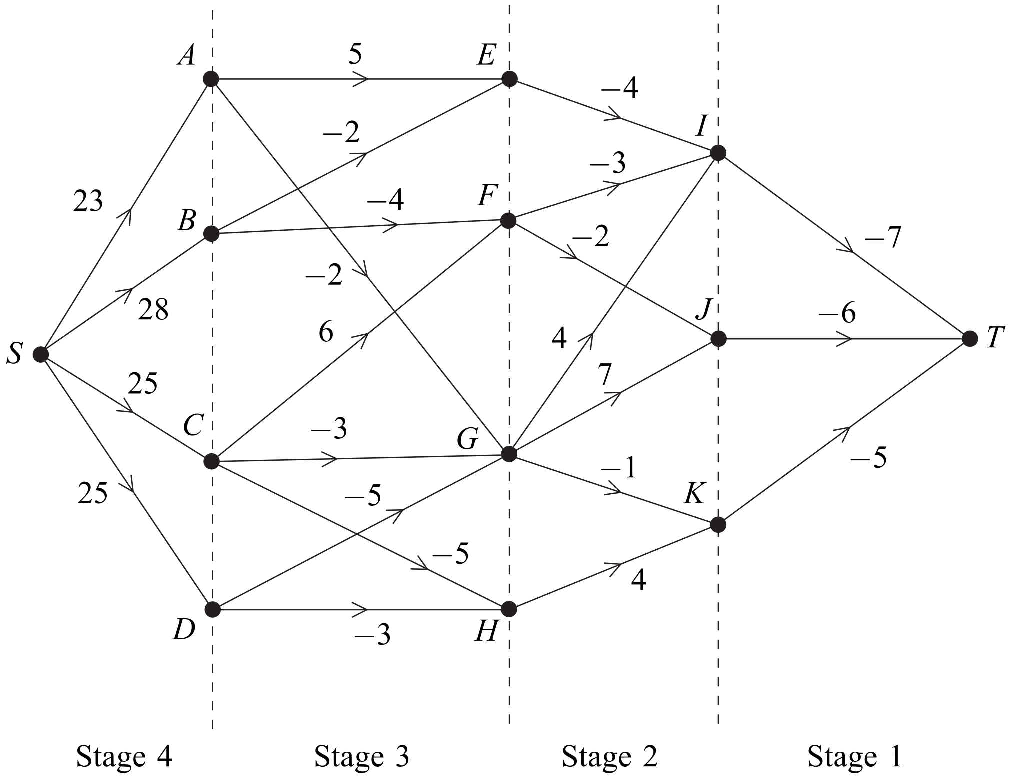
5 Each path from S to T in the network below represents a possible way of using the internet to buy a ticket for a particular event. The number on each edge represents a charge, in pounds, with a negative value representing a discount. For example, the path $SAEIT$ represents a ticket costing $23 + 5 - 4 - 7 = 17$ pounds.



(a) **By working backwards from *T*** and completing the table on **Figure 4**, use dynamic programming to find the minimum weight of all paths from S to T. *(6 marks)*

(b) State the minimum cost of a ticket for the event and the paths corresponding to this minimum cost. *(3 marks)*

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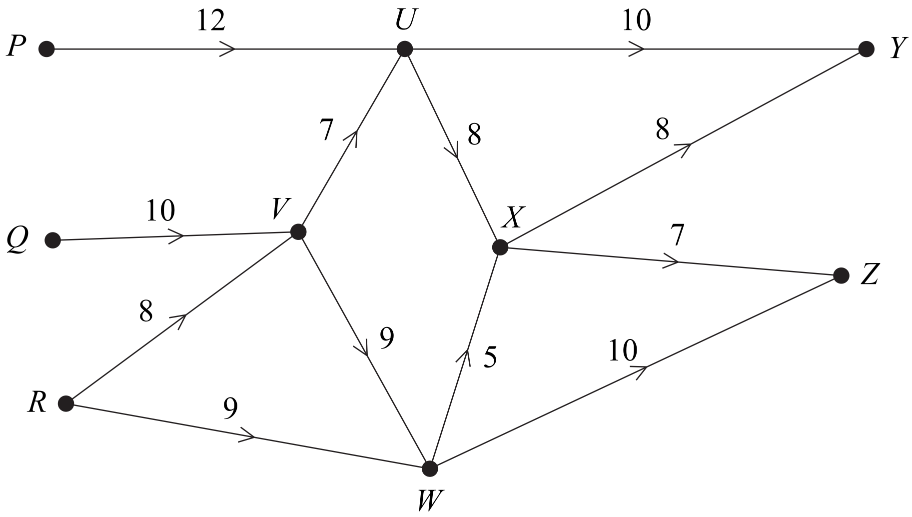
(a)

Figure 4

Stage	State	From	Value	
1	I	T	-7	
	J	T	-6	
	K	T	-5	
2	E	I	$-7-4=-11$	
	F	I		
		J		
	G	I		
		J		
		K		
	H	K		
3				

6 A retail company has warehouses at P, Q and R, and goods are to be transported to retail outlets at Y and Z. There are also retaining depots at U, V, W and X.

The possible routes with the capacities along each edge, in van loads per week, are shown in the following diagram.



(a) On **Figure 5 opposite**, add a super-source, S, and a super-sink, T, and appropriate edges so as to produce a directed network with a single source and a single sink. Indicate the capacity of each edge that you have added. *(2 marks)*

(b) On **Figure 6**, write down the maximum flows along the routes $SPUYT$ and $SRVWZT$. *(2 marks)*

(c) (i) On **Figure 7**, add the vertices S and T and the edges connecting S and T to the network. Using the maximum flows along the routes $SPUYT$ and $SRVWZT$ found in part **(b)** as the initial flow, indicate the potential increases and decreases of the flow on each edge of these routes. *(2 marks)*

(ii) Use flow augmentation to find the maximum flow from S to T. You should indicate any flow-augmenting routes on **Figure 6** and modify the potential increases and decreases of the flow on **Figure 7**. *(4 marks)*

(d) Find a cut with value equal to the maximum flow. *(1 mark)*

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Figure 5

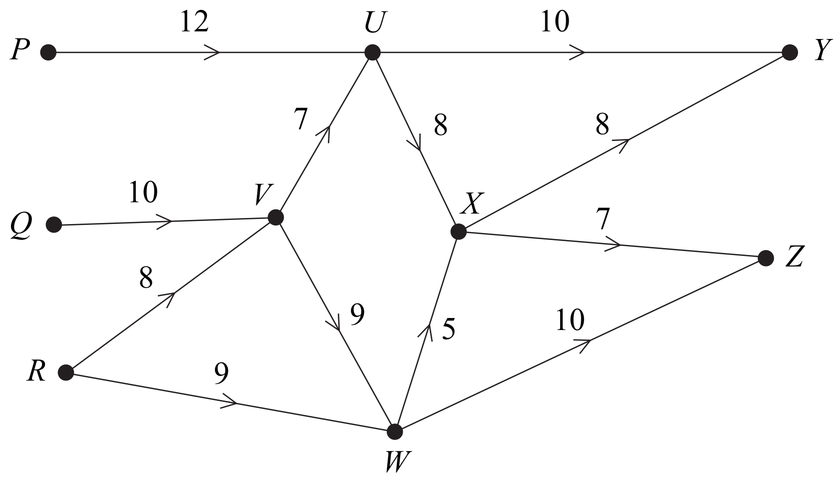
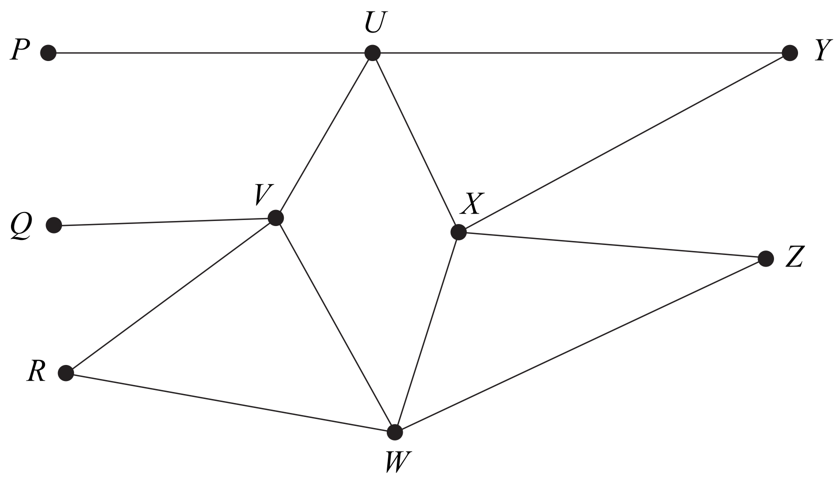


Figure 6

Route	Flow
SPUYT	
SRVWZT	

Figure 7



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END OF QUESTIONS