

4725

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## 4725 Further Pure Mathematics 1

1	$\frac{7}{26} + \frac{17}{26}i$	M1 A1 A1 A1	4 <b>4</b>	Multiply by conjugate of denominator Obtain correct numerator Obtain correct denominator
2	(i) $\frac{1}{10} \begin{pmatrix} 5 & 0 \\ -a & 2 \end{pmatrix}$ (ii) $\begin{pmatrix} 3 & -2 \\ 2a & 6 \end{pmatrix}$	B1 B1  B1 B1	2  2 <b>4</b>	Both diagonals correct Divide by correct determinant  Two elements correct Remaining elements correct
3	$n^2(n+1)^2 + n(n+1)(2n+1) + n(n+1)$ $n(n+1)^2(n+2)$	M1 A1 A1 M1 A1ft A1	6 <b>6</b>	Express as sum of 3 terms 2 correct unsimplified terms 3 <sup>rd</sup> correct unsimplified term Attempt to factorise Two factors found, ft their quartic Correct final answer a.e.f.
4	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	B1 M1 A1 A1	4 <b>4</b>	State or use correct result Combine matrix and its inverse Obtain <b>I</b> or <b>I</b> <sup>2</sup> but not 1 Obtain zero <b>matrix</b> but not 0 <b>S.C. If 0/4, B1 for <math>AA^{-1} = I</math></b>
5	<i>Either</i> $4k - 4$ $k = 1$  <i>Or</i>	M1 M1 A1 M1 A1ft  M1 A1 M1 A1 A1	5   <b>5</b>	Consider determinant of coefficients of LHS Sensible attempt at evaluating any $3 \times 3$ det Obtain correct answer a.e.f. unsimplified Equate det to 0 Obtain $k = 1$ , ft provided all M's awarded  Eliminate either $x$ or $y$ Obtain correct equation Eliminate 2 <sup>nd</sup> variable Obtain correct linear equation Deduce that $k = 1$
6	(i) <i>Either</i> <i>Or</i>  (ii) (iii) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (iv)	B1 DB1 B1 DB1  B1 DB1 B1 B1 B1B1B1	2  2 2 3 <b>9</b>	Reflection, in $x$ -axis Stretch parallel to $y$ -axis, s.f. $-1$  Reflection, in $y = -x$  Each column correct  Rotation, $90^\circ$ , clockwise about $O$ <b>S.C. If (iii) incorrect, B1 for identifying their transformation, B1 all details correct</b>

7	<p>(i) <math>13^n + 6^{n-1} + 13^{n+1} + 6^n</math></p> <p>(ii)</p>	<p>B1 M1 A1 B1 B1 B1 B1</p>	<p>3     4 <b>7</b></p>	<p>Correct expression seen Attempt to factorise both terms in (i) Obtain correct expression Check that result is true for <math>n=1</math> ( or 2) Recognise that (i) is divisible by 7 Deduce that <math>u_{n+1}</math> is divisible by 7 Clear statement of Induction conclusion</p>
8	<p>(i)</p> <p>(ii) <math>\alpha + \beta = 6k, \alpha\beta = k^2</math> <math>\alpha - \beta = (4\sqrt{2})k</math></p> <p>(iii) <math>\sum \alpha' = 6k</math> <math>\alpha' \beta' = \alpha\beta - (\alpha - \beta) - 1</math> <math>\alpha' \beta' = k^2 - (4\sqrt{2})k - 1</math> <math>x^2 - 6kx + k^2 - (4\sqrt{2})k - 1 = 0</math></p>	<p>M1 A1  B1 B1 M1 A1  B1ft  M1  A1ft  B1ft</p>	<p>2   4    4 <b>10</b></p>	<p>Expand at least 1 of the brackets Derive given answer correctly  State or use correct values Find value of <math>\alpha - \beta</math> using (i) Obtain given value correctly ( allow if <math>-6k</math> used )  Sum of new roots stated or used  Express new product in terms of old roots  Obtain correct value for new product  Write down correct quadratic <b>equation</b></p>
9	<p>(i)</p> <p>(ii)</p> <p><math>1 + \frac{1}{3} - \frac{1}{2n-1} - \frac{1}{2n+1}</math></p> <p>(iii) <math>\frac{4}{3}</math></p>	<p>M1 A1  M1 M1 A1 A1 M1 A1  B1ft</p>	<p>2          6  1 <b>9</b></p>	<p>Use correct denominator Obtain given answer correctly  Express terms as differences using (i) Do this for at least 1<sup>st</sup> 3 terms First 3 terms all correct Last 3 terms all correct ( in terms of <math>n</math> or <math>r</math>) Show pairs cancelling Obtain correct answer, a.e.f.( in terms of <math>n</math>)  Given answer deduced correctly, ft their (ii)</p>

<b>10</b>	<b>(i)</b> $x^2 - y^2 = 2, 2xy = \sqrt{5}$	M1 A1		Attempt to equate real and imaginary parts Obtain both results a.e.f.
	$4x^4 - 8x^2 - 5 = 0$	M1 M1		Eliminate to obtain quadratic in $x^2$ or $y^2$ Solve to obtain $x$ (or $y$ ) values
	$x = \pm \frac{\sqrt{10}}{2}, y = \pm \frac{\sqrt{2}}{2}$ $\pm (\frac{\sqrt{10}}{2} + i \frac{\sqrt{2}}{2})$	A1 A1	6	Correct values for both $x$ & $y$ obtained a.e.f. Correct answers as complex numbers
	<b>(ii)</b> $z^2 = 2 \pm i\sqrt{5}$ $z = \pm (\frac{\sqrt{10}}{2} \pm i \frac{\sqrt{2}}{2})$	M1 A1 M1 A1ft	4	Solve quadratic in $z^2$ Obtain correct answers Use results of (i) Obtain correct answers, ft must include root from conjugate
<b>(iii)</b>	B1ft	1	Sketch showing roots correctly	
<b>(iv)</b>	B1 B1ft B1ft	3	Sketch of straight line, $\perp$ to $\alpha$ Bisector	
		<b>14</b>		