Paper Reference(s)

6680

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

January 2005

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as Texas Instruments TI 89, TI 92, Casio 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the Answer Book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other names and signature.
Whenever a numerical value of g is required, take $g = 9.8\text{ms}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. [*In this question* $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal perpendicular unit vectors.*]

Two smooth uniform spheres A and B have equal radius but masses m and $5m$ respectively. The spheres are moving on a smooth horizontal plane when they collide. Immediately before the collision, the velocities of A and B are $(\mathbf{i}+2\mathbf{j})$ m s^{-1} and $(-\mathbf{i}+3\mathbf{j})$ m s^{-1} respectively. Immediately after the collision, the velocity of A is $(-2\mathbf{i}+5\mathbf{j})$ m s^{-1}.

(a) By considering the impulse on A, find a unit vector parallel to the line joining the centres of the spheres when they collide.

(4)

(b) Find the velocity of B immediately after the collision.

(3)

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2. [*In this question* $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal unit vectors due east and due north respectively.*]

A man cycling at a constant speed u on horizontal ground finds that, when his velocity is $u\mathbf{j}$ m s^{-1}, the velocity of the wind appears to be $v(3\mathbf{i}-4\mathbf{j})$ m s^{-1}, where v is a constant. When the velocity of the man is $\frac{u}{5}(-3\mathbf{i}+4\mathbf{j})$ m s^{-1}, he finds that the velocity of the wind appears to be $w\mathbf{i}$ m s^{-1}, where w is a constant.

(a) Show that $v=\frac{u}{20}$, and find w in terms of u.

(5)

(b) Find, in terms of u, the true velocity of the wind.

(2)

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3. Two ships A and B are sailing in the same direction at constant speeds of 12 km h^{-1} and 16 km h^{-1} respectively. They are sailing along parallel lines which are 4 km apart. When the distance between the ships is 4 km, B turns through 30° towards A.

Find the shortest distance between the ships in the subsequent motion.

(7)

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4. A car of mass M moves along a straight horizontal road. The total resistance to motion of the car is modelled as having constant magnitude R. The engine of the car works at a constant rate RU.

Find the time taken for the car to accelerate from a speed of $\frac{1}{4}U$ to a speed of $\frac{1}{2}U$.

(9)

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5. [*In this question* **i** *and* **j** *are horizontal perpendicular unit vectors.*]

The vector $\mathbf{n} = (-\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j})$ and the vector $\mathbf{p} = (-\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j})$ are perpendicular unit vectors.

(a) Verify that $\frac{9}{5}\mathbf{n} + \frac{13}{5}\mathbf{p} = (\mathbf{i} + 3\mathbf{j})$.

(2)

A smooth uniform sphere S of mass 0.5 kg is moving on a smooth horizontal plane when it collides with a fixed smooth vertical wall which is parallel to $\mathbf{p}$. Immediately after the collision the velocity of S is $(\mathbf{i} + 3\mathbf{j})$ m s^{-1}. The coefficient of restitution between S and the wall is $\frac{9}{16}$.

(b) Find, in terms of **i** and **j**, the velocity of S immediately before the collision.

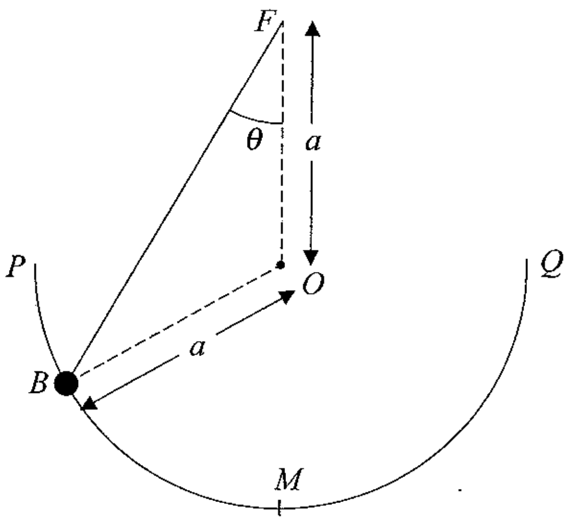
(5)

(c) Find the energy lost in the collision.

(3)

6.

Figure 1



A smooth wire PMQ is in the shape of a semicircle with centre O and radius a. The wire is fixed in a vertical plane with PQ horizontal and the mid-point M of the wire vertically below O. A smooth bead B of mass m is threaded on the wire and is attached to one end of a light elastic string. The string has modulus of elasticity $4mg$ and natural length $\frac{5}{4}a$. The other end of the string is attached to a fixed point F which is a distance a vertically above O, as shown in Fig. 1.

(a) Show that, when $\angle BFO = \theta$, the potential energy of the system is

$$\tfrac{1}{10}mga(8\cos\theta - 5)^2 - 2mga\cos^2\theta + \text{constant}.$$

(6)

(b) Hence find the values of θ for which the system is in equilibrium.

(6)

(c) Determine the nature of the equilibrium at each of these positions.

(5)

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7. A particle of mass m is attached to one end P of a light elastic spring PQ, of natural length a and modulus of elasticity man^2. At time $t = 0$, the particle and the spring are at rest on a smooth horizontal table, with the spring straight but unstretched and uncompressed. The end Q of the spring is then moved in a straight line, in the direction PQ, with constant acceleration f. At time t, the displacement of the particle in the direction PQ from its initial position is x and the length of the spring is $(a + y)$.

(a) Show that $x + y = \frac{1}{2}ft^2$.

(2)

(b) Hence show that

$$\frac{d^2x}{dt^2} + n^2x = \frac{1}{2}n^2ft^2.$$

(6)

You are given that the general solution of this differential equation is

$$x = A\cos nt + B\sin nt + \frac{1}{2}ft^2 - \frac{f}{n^2},$$

where A and B are constants.

(c) Find the values of A and B.

(6)

(d) Find the maximum tension in the spring.

(4)

END