



ADVANCED GCE
MATHEMATICS
Further Pure Mathematics 2

4726

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

None

Monday 11 January 2010
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

2

1 It is given that $f(x) = x^2 - \sin x$.

(i) The iteration $x_{n+1} = \sqrt{\sin x_n}$, with $x_1 = 0.875$, is to be used to find a real root, α , of the equation $f(x) = 0$. Find x_2 , x_3 and x_4 , giving the answers correct to 6 decimal places. [2]

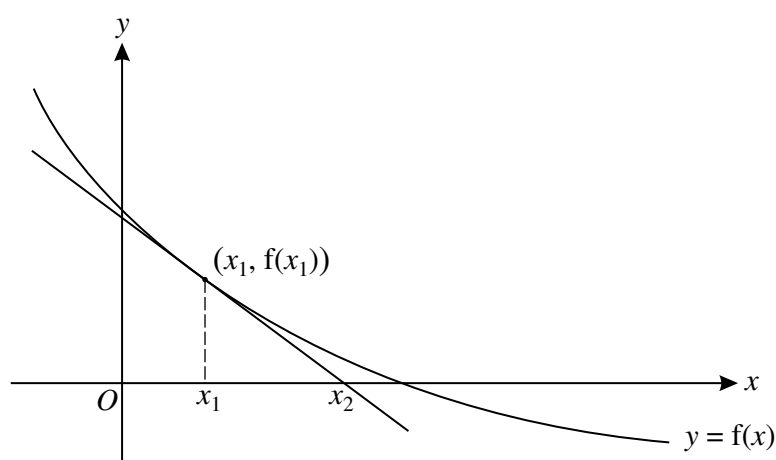
(ii) The error e_n is defined by $e_n = \alpha - x_n$. Given that $\alpha = 0.876726$, correct to 6 decimal places, find e_3 and e_4 . Given that $g(x) = \sqrt{\sin x}$, use e_3 and e_4 to estimate $g'(\alpha)$. [3]

2 It is given that $f(x) = \tan^{-1}(1 + x)$.

(i) Find $f(0)$ and $f'(0)$, and show that $f''(0) = -\frac{1}{2}$. [4]

(ii) Hence find the Maclaurin series for $f(x)$ up to and including the term in x^2 . [2]

3



A curve with no stationary points has equation $y = f(x)$. The equation $f(x) = 0$ has one real root α , and the Newton-Raphson method is to be used to find α . The tangent to the curve at the point $(x_1, f(x_1))$ meets the x -axis where $x = x_2$ (see diagram).

(i) Show that $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$. [3]

(ii) Describe briefly, with the help of a sketch, how the Newton-Raphson method, using an initial approximation $x = x_1$, gives a sequence of approximations approaching α . [2]

(iii) Use the Newton-Raphson method, with a first approximation of 1, to find a second approximation to the root of $x^2 - 2 \sinh x + 2 = 0$. [2]

4 The equation of a curve, in polar coordinates, is

$$r = e^{-2\theta}, \quad \text{for } 0 \leq \theta \leq \pi.$$

(i) Sketch the curve, stating the polar coordinates of the point at which r takes its greatest value. [2]

(ii) The pole is O and points P and Q , with polar coordinates (r_1, θ_1) and (r_2, θ_2) respectively, lie on the curve. Given that $\theta_2 > \theta_1$, show that the area of the region enclosed by the curve and the lines OP and OQ can be expressed as $k(r_1^2 - r_2^2)$, where k is a constant to be found. [5]

3

- 5 (i) Using the definitions of $\sinh x$ and $\cosh x$ in terms of e^x and e^{-x} , show that

$$\cosh^2 x - \sinh^2 x \equiv 1.$$

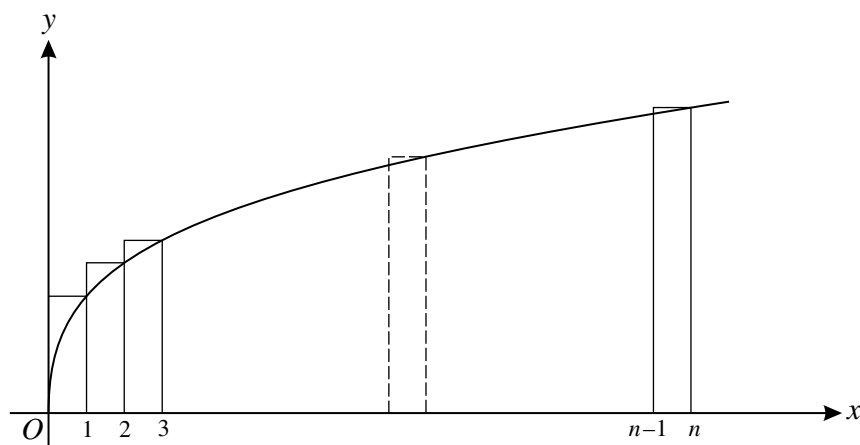
Deduce that $1 - \tanh^2 x \equiv \operatorname{sech}^2 x$. [4]

- (ii) Solve the equation $2 \tanh^2 x - \operatorname{sech} x = 1$, giving your answer(s) in logarithmic form. [4]

- 6 (i) Express $\frac{4}{(1-x)(1+x)(1+x^2)}$ in partial fractions. [5]

- (ii) Show that $\int_0^{\frac{1}{\sqrt{3}}} \frac{4}{1-x^4} dx = \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{1}{3}\pi$. [4]

7



The diagram shows the curve with equation $y = \sqrt[3]{x}$, together with a set of n rectangles of unit width.

- (i) By considering the areas of these rectangles, explain why

$$\sqrt[3]{1} + \sqrt[3]{2} + \sqrt[3]{3} + \dots + \sqrt[3]{n} > \int_0^n \sqrt[3]{x} dx. \quad [2]$$

- (ii) By drawing another set of rectangles and considering their areas, show that

$$\sqrt[3]{1} + \sqrt[3]{2} + \sqrt[3]{3} + \dots + \sqrt[3]{n} < \int_1^{n+1} \sqrt[3]{x} dx. \quad [3]$$

- (iii) Hence find an approximation to $\sum_{n=1}^{100} \sqrt[3]{n}$, giving your answer correct to 2 significant figures. [3]

[Questions 8 and 9 are printed overleaf.]

4

8 The equation of a curve is

$$y = \frac{kx}{(x-1)^2},$$

where k is a positive constant.

(i) Write down the equations of the asymptotes of the curve. [2]

(ii) Show that $y \geq -\frac{1}{4}k$. [4]

(iii) Show that the x -coordinate of the stationary point of the curve is independent of k , and sketch the curve. [4]

9 (i) Given that $y = \tanh^{-1}x$, for $-1 < x < 1$, prove that $y = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$. [3]

(ii) It is given that $f(x) = a \cosh x - b \sinh x$, where a and b are positive constants.

(a) Given that $b \geq a$, show that the curve with equation $y = f(x)$ has no stationary points. [3]

(b) In the case where $a > 1$ and $b = 1$, show that $f(x)$ has a minimum value of $\sqrt{a^2 - 1}$. [6]

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.