

# **Mark Scheme 4724**

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<b>1</b>	$\frac{d}{dx}(xy) = x \frac{dy}{dx} + y$ $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$ <p>Substitute (1,2) into their differentiated equation and attempt to solve for <math>\frac{dy}{dx}</math>. [Allow subst of (2,1)]</p> $\frac{dy}{dx} = -2$	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b> dep at least 1 x <b>B1</b></p> <p><b>A1</b></p>	<p>s.o.i. e.g. <math>2x \frac{dy}{dx} + y</math></p> <p>Or attempt to solve their diff equation for <math>\frac{dy}{dx}</math> and then substitute (1,2)</p> <p><b>4</b></p>
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<b>2</b>	<p>(i) <math>1 + (-2)(-3x) + \frac{(-2)(-3)}{1.2}(-3x)^2 (+ \dots \text{ignore})</math></p> <p><math>= 1 + 6x</math></p> <p><math>\dots + 27x^2</math></p> <p>(ii) <math>(1 + 2x)^2(1 - 3x)^{-2}</math></p> <p>Attempt to expand <math>(1 + 2x)^2</math> &amp; select (at least) 2 relevant products and add</p> <p>55 (Accept <math>55x^2</math>)</p> <p><u>SR 1</u> For expansion of <math>(1 + 2x)^2</math> with 1 error, A1√</p> <p><u>SR 2</u> For expansion of <math>(1 + 2x)^2</math> &amp; &gt; 1 error, A0</p> <p><b>Alternative Method</b></p> <p>For correct method idea of long division</p> <p>1 ..... +10x ..... +55x<sup>2</sup></p>	<p><b>M1</b></p> <p><b>B1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A2</b>√</p> <p><b>M1</b></p> <p><b>A1,A1,A1(4)</b></p>	<p>State or imply; accept <math>-3x^2</math> &amp; <math>-9x^2</math></p> <p>Correct first 2 terms</p> <p>Correct third term</p> <p>For changing into suitable form, seen/implicit</p> <p>Selection may be after multiplying out</p> <p>4 If (i) is <math>a + bx + cx^2</math>, f.t. <math>4(a + b) + c</math></p>
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<b>3</b>	<p>(i) <math>\frac{A}{x} + \frac{B}{3-x}</math> &amp; c-u rule or <math>A(3-x) + Bx \equiv 3 - 2x</math></p> <p><math>\frac{1}{x}</math></p> <p><math>-\frac{1}{3-x}</math></p> <p>(ii) <math>\int \frac{1}{x}(dx) = \ln x</math> or <math>\ln x </math></p> <p><math>\int \frac{1}{3-x}(dx) = -\ln(3-x)</math> or <math>-\ln 3-x </math></p> <p>Correct method idea of substitution of limits</p> <p><math>\ln 2 (+ \ln 1 - \ln 1) - \ln 2 = 0</math></p> <p><b>Alternative Method</b></p> <p>If ignoring PFs, <math>\ln x(3-x)</math> immediately</p> <p>As before</p> <p>(iii) Suitable statement or clear implication e.g. Equal amounts (of area) above and below (axis) or graph crosses axis or there's a root (Be lenient)</p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>B2</b></p> <p><b>M1,A1 (4)</b></p> <p><b>B1</b></p>	<p>Correct format + suitable method</p> <p>seen in (i) or (ii)</p> <p>3 ditto; <math>\frac{1}{x} - \frac{1}{3-x}</math> scores 3 immediately</p> <p>Check sign carefully; do not allow <math>\ln(x-3)</math></p> <p>Dep on an attempt at integrating</p> <p>Clearly seen; WWW <b>AG</b></p> <p><math>\ln x(x-3) \rightarrow 0</math></p> <p><b>1</b></p>

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4	(i) Working out $\mathbf{b} - \mathbf{a}$ or $\mathbf{a} - \mathbf{b}$ or $\mathbf{c} - \mathbf{a}$ or $\mathbf{a} - \mathbf{c}$	<b>M1</b>	) Irrespective of label
	= $\pm(-3\mathbf{i} - \mathbf{j} - \mathbf{k})$ or $\pm(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	<b>A1</b>	) If not scored ,these 1 <sup>st</sup> 3 marks can be
	Method for finding magnitude of <u>any</u> vector	<b>M1</b>	) awarded in part (ii)
	Method for finding scalar product of <u>any</u> 2 vectors	<b>M1</b>	
	Using $\cos \theta = \frac{a \cdot b}{ a  b }$ AEF for <u>any</u> 2 vectors	<b>M1</b>	
	<u>[Alternative cosine rule method]</u> $ \vec{BC}  = \sqrt{6}$	<b>B1</b>	
Cosine rule used	<b>M1</b>	‘Recognisable’ form	
45.3°, 0.79(0), $\frac{\pi}{3.97}$ (45.289378, 0.7904487)	<b>A1</b>	<b>6</b> Do not accept supplement (134.7 etc)	

(ii)	Use of $\frac{1}{2}  \vec{AB}   \vec{AC}  \sin \theta$	<b>M1</b>	Accept $\left  \frac{1}{2} \vec{AB} \times \vec{AC} \right $
	3.54 (3.5355) or $\frac{5\sqrt{2}}{2}$	<b>A1</b>	<b>2</b> Accept from correct supp (134.7 etc)

5	(i) $\frac{dA}{dt}$ or $kA^2$ seen	<b>M1</b>	
	$\frac{dA}{dt} = kA^2$	<b>A1</b>	<b>2</b>

(ii)	Separate variables + attempt to integrate	<b>*M1</b>	Accept if based on $\frac{dA}{dt} = kA^2$ or $A^2$
	$-\frac{1}{A} = kt + c$ or $-\frac{1}{kA} = t + c$ or $-\frac{1}{A} = t + c$	<b>A1</b>	
	Subst one of (0,0),(1,1000) or (2,2000) into eqn.	<b>dep*M1</b>	Equation must contain $k$ and/or $c$
	Subst another of (0,),(1,1000) or (2,2000) into eqn	<b>dep*M1</b>	This equation must contain $k$ <u>and</u> $c$
	Substitute $A = 3000$ into eqn with $k$ and $c$ subst	<b>dep*M1</b>	
	$t = \frac{7}{3}$ ISW	<b>A1</b>	<b>6</b> Accept 2.33, 2h 20 m

6	(i) Attempt to connect $du$ and $dx$ e.g. $\frac{du}{dx} = e^x$	<b>M1</b>	But not $du = dx$
	Use of $e^{2x} = (e^x)^2$ or $(u-1)^2$ s.o.i.	<b>A1</b>	
	Simplification to $\int \frac{u-1}{u} (du)$ WWW	<b>A1</b>	<b>3 AG</b>

(ii)	Change $\frac{u-1}{u}$ to $1 - \frac{1}{u}$ or use parts	<b>M1</b>	If parts, may be twice if $\int \ln x dx$ is involved
	$\int \frac{1}{u} du = \ln u$	<b>A1</b>	Seen anywhere in this part
	<u>Either</u> attempt to change limits <u>or</u> resubstitute	<b>M1</b> (indep)	Expect new limits $e+1$ & $2$
	Show as $e+1 - \ln(e+1) - \{2 \text{ or } (1+1)\} + \ln 2$	<b>A1</b>	
	WWW show final result as $e-1 - \ln\left(\frac{e+1}{2}\right)$	<b>A1</b>	<b>5 AG</b>

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7	<p>(i) Produce at least 2 of the 3 relevant eqns in <math>\lambda</math> and <math>\mu</math> <b>M1</b>                  Solve the 2 eqns in <math>\lambda</math> &amp; <math>\mu</math> as far as <math>\lambda = \dots</math> or <math>\mu = \dots</math> <b>M1</b>                  1<sup>st</sup> solution: <math>\lambda = -2</math> or <math>\mu = 3</math> <b>A1</b>                  2<sup>nd</sup> solution: <math>\mu = 3</math> or <math>\lambda = -2</math> f.t. <b>A1</b>✓                  Substitute their <math>\lambda</math> and <math>\mu</math> into 3<sup>rd</sup> eqn and find ‘<math>a</math>’ <b>M1</b>                  Obtain <math>a = 2</math> &amp; clearly state that <math>a</math> cannot be 2 <b>A1</b></p> <p>(ii) Subst their <math>\lambda</math> or <math>\mu</math> (&amp; poss <math>a</math>) into either line eqn <b>M1</b>                  Point of intersection is <math>-5\mathbf{i} - 4\mathbf{j}</math> <b>A1</b></p> <p><b>N.B.</b> In this question, award marks irrespective of labelling of parts</p>	<p>e.g. <math>1 + 3\lambda = -8 + \mu</math>, <math>-2 + \lambda = 2 - 2\mu</math></p> <p><b>6</b></p> <p>2 Accept any format <u>No f.t. here</u></p>
8	<p>(i) <u>Integration method</u>                  Attempt to change <math>\cos^2 6x</math> into <math>f(\cos 12x)</math> <b>M1</b>  <math>\cos^2 6x = \frac{1}{2}(1 + \cos 12x)</math> <b>A1</b>  <math>\int = \frac{1}{2}x + \frac{1}{24}\sin 12x + c</math> <b>A1</b></p> <p><u>Differentiation method</u>                  Differentiate RHS producing <math>\frac{1}{2} + \frac{1}{2}\cos 12x</math> ---(E) <b>B1</b>                  Attempt to change <math>\cos 12x</math> into <math>f(\cos 6x)</math> <b>M1</b>                  Simplify (E) WWW to <math>\cos^2 6x</math> + satis finish <b>A1</b></p>	<p>with <math>\cos^2 6x</math> as the subject of the formula  <b>AG</b> Accept <math>\frac{1}{2}(x + \frac{1}{12}\sin 12x)</math></p> <p>Accept <math>+/- 2\cos^2 6x + +/- 1</math></p> <p><b>3</b></p>
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	<p>(ii) Parts with <math>u = x</math>, <math>dv = \cos^2 6x</math> <b>*M1</b>  <math>x(\frac{1}{2}x + \frac{1}{24}\sin 12x) - \int(\frac{1}{2}x + \frac{1}{24}\sin 12x)dx</math> <b>A1</b>  <math>\int \sin 12x dx = -\frac{1}{12}\cos 12x</math> <b>B1</b>                  Correct use of limits to <u>whole</u> integral <b>dep*M1</b>  <math>\frac{\pi^2}{288} - \frac{\pi^2}{576} - \frac{1}{288} - \frac{1}{288}</math> <b>A1</b>  <math>\frac{\pi^2}{576} - \frac{1}{144}</math> <b>+A1</b></p> <p>S.R. If final marks are A0 + A0, allow SR A1 for</p>	<p>Correct expression only</p> <p>Clear indication somewhere in this part</p> <p>Accept ( ) (-0)</p> <p>AE unsimp exp. Accept <math>12x24, \sin \pi</math> here</p> <p><b>6</b> Tolerate e.g. <math>\frac{2}{288}</math> here                  0.01/0.010/0.0101/0.0102/0.0101902</p>

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9	<p>(i) <math>\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}</math></p> <p><math>\frac{dx}{dt} = -4 \sin t</math> or <math>\frac{dy}{dt} = 3 \cos t</math></p> <p><math>\frac{dy}{dx} = -\frac{3 \cos t}{4 \sin t}</math> or <math>\frac{3 \cos t}{-4 \sin t}</math> ISW</p> <p>SR: M1 for Cartesian eqn attempt + B1 for <math>\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}</math> + A1 as before (must be in terms of <math>t</math>)</p>	M1	Used, not just quoted
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(ii)	<p><math>y - 3 \sin p = \left( \text{their } \frac{dy}{dx} \right) (x - 4 \cos p)</math></p> <p>or <math>y = \left( \text{their } \frac{dy}{dx} \right) x + c</math> &amp; subst cords to find <math>c</math></p> <p><math>4y \sin p - 12 \sin^2 p = -3x \cos p + 12 \cos^2 p</math></p> <p>or <math>c = \frac{12 \sin^2 p + 12 \cos^2 p}{4 \sin p}</math></p> <p><math>3x \cos p + 4y \sin p = 12</math> WWW</p>	M1	Accept $p$ or $t$ here
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(iii)	<p>Subst <math>x = 0</math> and <math>y = 0</math> separately in tangent eqn</p> <p>Produce <math>\frac{3}{\sin p}</math> and <math>\frac{4}{\cos p}</math></p> <p>Use <math>\Delta = \frac{1}{2} \left( \frac{3}{\sin p} \cdot \frac{4}{\cos p} \right) = \frac{12}{\sin 2p}</math> WWW</p>	M1	to find $R$ & $S$
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(iv)	<p>Least area = 12</p> <p><math>p = \frac{1}{4}\pi</math> as final or only answer</p> <p>S.R. <math>45^\circ \rightarrow B1</math> ;</p>	B1	<p>Accept <math>\frac{12}{4 \sin p}</math> and/or <math>\frac{12}{3 \cos p}</math></p> <p>3 AG</p> <p>3 These B marks are independent.</p> <p>S.R. <math>[-12</math> and e.g. <math>-\pi/4 \rightarrow B1]</math></p>

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