

Version 1.0



**General Certificate of Education (A-level)**  
**June 2011**

**Mathematics**

**MPC3**

**(Specification 6360)**

**Pure Core 3**

**Final**

***Mark Scheme***

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## Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MPC3 – June 2011**

Q	Solution	Marks	Total	Comments																
1 (a)	$\frac{1}{6}$ or $\left(\frac{1}{6}, 0\right)$	B1	1	condone 0.167 AWRT																
1 (b)	$\left(\frac{dy}{dx}\right) = \frac{1}{x}$	M1 A1	2	$\frac{k}{x}$ where $k = 1, 6$ or $\frac{1}{6}$ $k = 1$																
1 (c)	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="border-right: 1px solid black; border-bottom: 1px solid black;">x</th> <th style="border-bottom: 1px solid black;">y</th> </tr> </thead> <tbody> <tr><td style="border-right: 1px solid black;">1</td><td>ln 6 = 1.7918</td></tr> <tr><td style="border-right: 1px solid black;">2</td><td>ln 12 = 2.4849</td></tr> <tr><td style="border-right: 1px solid black;">3</td><td>ln 18 = 2.8904</td></tr> <tr><td style="border-right: 1px solid black;">4</td><td>ln 24 = 3.1781</td></tr> <tr><td style="border-right: 1px solid black;">5</td><td>ln 30 = 3.4012</td></tr> <tr><td style="border-right: 1px solid black;">6</td><td>ln 36 = 3.5835</td></tr> <tr><td style="border-right: 1px solid black;">7</td><td>ln 42 = 3.7377</td></tr> </tbody> </table> $A = \frac{1}{3} \times 1 \left[ (1.7918 + 3.7377) + 4(2.4849 + 3.1781 + 3.5835) + 2(2.8904 + 3.4012) \right]$ <p style="text-align: right;">= 18.4</p>	x	y	1	ln 6 = 1.7918	2	ln 12 = 2.4849	3	ln 18 = 2.8904	4	ln 24 = 3.1781	5	ln 30 = 3.4012	6	ln 36 = 3.5835	7	ln 42 = 3.7377	M1  A1  M1 A1	4	5+ y-values correct, either exact or correct to 3SF (rounded or truncated) or better  all 7 y-values correct (and only these 7 values), either exact or correct to 3SF (rounded or truncated) or better  correct use of Simpson's rule on their 7 y-values, condone missing square brackets CAO this value only
x	y																			
1	ln 6 = 1.7918																			
2	ln 12 = 2.4849																			
3	ln 18 = 2.8904																			
4	ln 24 = 3.1781																			
5	ln 30 = 3.4012																			
6	ln 36 = 3.5835																			
7	ln 42 = 3.7377																			
	<b>Total</b>		<b>7</b>																	

**MPC3 (cont)**

Q	Solution	Marks	Total	Comments
2(a)(i)	$y = xe^{2x}$ $\left(\frac{dy}{dx} = \right) 2xe^{2x} + e^{2x}$	M1 A1 A1 ISW	3	$kxe^{2x} + le^{2x}$ where $k$ and $l$ are 1s or 2s $k = 2$ $l = 1$ } Independent of each other $(= e^{2x}(2x+1))$
(ii)	$x = 1 \Rightarrow \frac{dy}{dx} = 3e^2$ tangent: $y - e^2 = 3e^2(x - 1)$ OE	M1  A1	2	correct substitution of $x = 1$ into their $\frac{dy}{dx}$ but must have earned M1 in part (i) CSO (no ISW), must have scored first 4 marks common correct answer: $y = 3e^2x - 2e^2$
(b)	$y = \frac{2 \sin 3x}{1 + \cos 3x}$ $\left(\frac{dy}{dx} = \right) \frac{(1 + \cos 3x)6 \cos 3x - 2 \sin 3x(-3 \sin 3x)}{(1 + \cos 3x)^2}$ $= \frac{6 \cos 3x + 6 \cos^2 3x + 6 \sin^2 3x}{(1 + \cos 3x)^2}$ $= \frac{6 \cos 3x + 6}{(1 + \cos 3x)^2}$ $= \frac{6}{1 + \cos 3x}$	M1  A1  m1  A1	4	$\frac{\pm p(1 + \cos 3x) \cos 3x \pm q \sin 3x(\sin 3x)}{(1 + \cos 3x)^2}$ where $p$ and $q$ are rational numbers condone poor use/omission of brackets PI by further working this line must be seen in this form (ie in terms of $\cos^2 3x$ and $\sin^2 3x$ ), but allow $\sin^2 3x$ replaced by $1 - \cos^2 3x$ condone denominator correctly expanded correct use of $k \sin^2 3x + k \cos^2 3x = k$ or $k \sin^2 3x = k(1 - \cos^2 3x)$ CSO
<b>Total</b>			<b>9</b>	

**MPC3 (cont)**

Q	Solution	Marks	Total	Comments
<p><b>3(a)</b></p>	<p>note: if degrees used then no marks in (a) and (c)</p> $f(x) = \cos^{-1}(2x-1) - e^x$ $\left. \begin{aligned} f(0.4) &= 0.3 \\ f(0.5) &= -0.1 \end{aligned} \right\}$ <p>change of sign <math>\therefore 0.4 &lt; \alpha &lt; 0.5</math></p>	<p>M1</p> <p>A1</p> <p>(M1)</p> <p>(A1)</p>	<p>2</p>	<p>or reverse</p> <p>sight of <math>\pm 0.3</math> (AWRT) <b>AND</b> <math>\mp 0.1</math> (AWRT)</p> <p>CSO, note <math>f(x)</math> must be defined, condone <math>0.4 \leq \alpha \leq 0.5</math></p> <p>alternative method</p> $\left. \begin{aligned} e^{0.4} &= 1.5, \cos^{-1}(2 \times 0.4 - 1) = 1.8 \\ e^{0.5} &= 1.65, \cos^{-1}(2 \times 0.5 - 1) = 1.57 \end{aligned} \right\}$ <p>at <math>0.4 \ e^x &lt; \cos^{-1}(2x-1)</math></p> <p>at <math>0.5 \ e^x &gt; \cos^{-1}(2x-1)</math></p> <p><math>\therefore 0.4 &lt; \alpha &lt; 0.5</math></p>
<p><b>(b)</b></p>	$\cos^{-1}(2x-1) = e^x$ $2x-1 = \cos(e^x)$ $x = \frac{1}{2}(\cos(e^x) + 1) = \frac{1}{2} + \frac{1}{2}\cos(e^x)$	<p>B1</p>	<p>1</p>	<p>AG</p> <p>must see middle line, and no errors seen, but condone <math>\cos e^x</math></p>
<p><b>(c)</b></p>	$x_1 = 0.4$ $x_2 = 0.539$ $x_3 = 0.428$	<p>B1</p> <p>B1</p>	<p>2</p>	<p>CAO</p> <p>CAO</p>
<b>Total</b>			<b>5</b>	

**MPC3 (cont)**

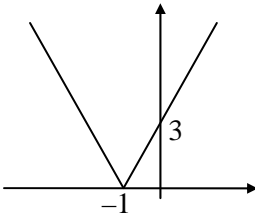
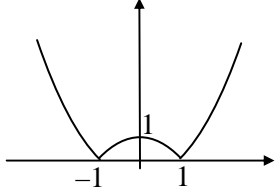
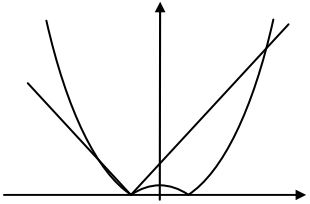
Q	Solution	Marks	Total	Comments
<p><b>4(a)(i)</b></p>	$(\sin^{-1} \pm 0.25) \pm 14.5$ $\theta = 194.5, 345.5$ (AWRT)	<p>M1 A1</p>	<p>2</p>	<p>PI by sight of 194.5 etc condone <math>\pm 14.4</math> no extras in interval, ignore answers outside interval</p>
<p><b>(ii)</b></p>	$2 \cot^2(2x + 30) = 2 - 7 \operatorname{cosec}(2x + 30)$ $2(\operatorname{cosec}^2(2x + 30) - 1) = 2 - 7 \operatorname{cosec}(2x + 30)$ $2 \operatorname{cosec}^2(2x + 30) + 7 \operatorname{cosec}(2x + 30) - 4 (= 0)$ $(2 \operatorname{cosec}(2x + 30) \pm 1)(\operatorname{cosec}(2x + 30) \pm 4) (= 0)$ $\operatorname{cosec}(2x + 30) = \frac{1}{2}$ or $-4$ $2x + 30 = 194.5, 345.5$ $x = 82.2, 157.8$ (AWRT)	<p>M1 A1 m1 A1</p>	<p>6</p>	<p>condone replacing <math>2x + 30</math> by <math>Y</math> correct use of <math>\operatorname{cosec}^2 Y = 1 + \cot^2 Y</math> must be in this form attempt at factorisation must be this line using <math>f(2x + 30)</math></p>
<p><b>(b)</b></p>	<p>stretch (I) scale factor <math>\frac{1}{2}</math> (II) parallel to <math>x</math>-axis (III) translate <math>\begin{pmatrix} -15 \\ 0 \end{pmatrix}</math> alternative method translate <math>\begin{pmatrix} -30 \\ 0 \end{pmatrix}</math> stretch scale factor <math>\frac{1}{2}</math> parallel to <math>x</math>-axis</p>	<p>B1 B1  M1 A1 E1 B1  (E1) (B1) (M1)  (A1)</p>	<p>4</p>	<p>one correct answer, allow 82.3, ignore extra solutions CAO both answers correct and no extras in interval, ignore answers outside interval  I and either II or III I + II + III  condone '15 to left' or '-15 in <math>x</math> (direction)'  as above  as above</p>
	<b>Total</b>		<b>12</b>	

**MPC3 (cont)**

Q	Solution	Marks	Total	Comments
5(a)	$[f(x)]$ not $1-1$	E1	1	OE
(b)	$y = \frac{1}{2x+1}$	M1	3	swap $x$ and $y$ } a correct next line } either order $[y =] \frac{1}{2} \left( \frac{1}{x} - 1 \right)$
	$x = \frac{1}{2y+1}$	M1		
	$2y+1 = \frac{1}{x}$	A1		
(c)	$[g^{-1}(x)] \neq -0.5$	B1	1	sight of $\neq -0.5$ OE
(d)	$\left( \frac{1}{2x+1} \right)^2 = \frac{1}{2x+1}$	B1	3	sight of $\left( \frac{1}{2x+1} \right)^2$ or $\frac{1}{(2x+1)^2}$ one correct step, must be one of these four lines
	$(2x+1) = (2x+1)^2$	M1		
	or $2x+1 = 4x^2 + 4x + 1$			
	or $\frac{1}{2x+1} = 1$			
	or $2x+1 = 1$	A1		CSO
	or $x = 0$			
<b>Total</b>			<b>8</b>	
6(a)	$3 \ln x = 4$ $\left( \ln x = \frac{4}{3} \right)$ $x = e^{\frac{4}{3}}$	B1	1	ISW. Condone $\sqrt[3]{e^4}$
(b)	$3 \ln x + \frac{20}{\ln x} = 19$	M1	5	correctly multiplying by $\ln x$ . use of formula, or completing the square must be correct condone $\sqrt[3]{e^4}$
	$3(\ln x)^2 + 20 = 19 \ln x$	A1		
	$3(\ln x)^2 - 19 \ln x + 20 (= 0)$	m1		
	$(3 \ln x \pm 4)(\ln x \pm 5) (= 0)$	A1		
	$\ln x = \frac{4}{3}, 5$	A1		
	$x = e^{\frac{4}{3}}, e^5$			
<b>Total</b>			<b>6</b>	



**MPC3 (cont)**

Q	Solution	Marks	Total	Comments
7(a)(i)		M1 A1	2	modulus graph, approximate V shape, touching negative $x$ -axis and crossing $y$ -axis -1, 3 marked, graph symmetrical, straight lines
(ii)		M1 A1 A1	3	modulus graph in 3 sections, touching $x$ -axis and crossing positive $y$ -axis correct curvature their $x > 1$ , their $x < -1$ } independent correct curve $-1 \leq x \leq 1$ and $x = \pm 1, y = 1$ marked
(b)(i)	$ 3x+3  =  x^2 - 1 $ $(3x+3 = x^2 - 1)$ $(0 =) x^2 - 3x - 4 \quad \text{---A}$ $x = 4, -1$ $(3x+3 = 1 - x^2)$ $x^2 + 3x + 2 (= 0) \quad \text{---B}$ $x = -1, -2$	M1 A1,A1  A1,A1	5	either A or B seen, all terms on one side  $\therefore x = -2, -1, 4$ SC NMS or partial method 1 correct value 1/5 } independent of 2 correct values 2/5 } method mark 3 correct values 5/5 } more than 3 distinct values max 2/5
(ii)	 <p><math>x &gt; 4, x &lt; -2</math></p>	M1,A1	2	$x >$ their largest, $x <$ their smallest; CAO
<b>Total</b>			<b>12</b>	

**MPC3 (cont)**

Q	Solution	Marks	Total	Comments
8	$\int \frac{1}{\cos^2 x (1 + 2 \tan x)^2} dx$ $u = 1 + 2 \tan x$ $\left(\frac{du}{dx} = 2 \sec^2 x\right) \text{ OE}$ $\int = \int \frac{du}{2u^2}$ $= \frac{1}{2} u^{-1}$ $= -\frac{1}{2u}$ $= -\frac{1}{2(1 + 2 \tan x)} (+c)$	<p>M1</p> <p>m1</p> <p>A1</p> <p>A1F</p> <p>A1</p>	<p>5</p>	<p>condone <math>\left(\frac{du}{dx} = a\right) \sec^2 x</math> where <math>a</math> is a constant</p> <p><math>\int \frac{k}{u^2} (du)</math>, where <math>k</math> is a constant</p> <p>correct, or <math>\frac{1}{2} \int u^{-2} (du)</math></p> <p>correct integral of their expression but must have scored M1 m1</p> <p>CSO, no ISW</p>
<b>Total</b>			<b>5</b>	

**MPC3 (cont)**

Q	Solution	Marks	Total	Comments
<p><b>9 (a)</b></p>	$\int x \ln x \, dx$ $\left. \begin{aligned} u = \ln x & \quad \frac{dv}{(dx)} = x \\ \frac{du}{(dx)} = \frac{1}{x} & \quad v = \frac{x^2}{2} \end{aligned} \right\}$ $\int = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{1}{x} (dx)$ $= \frac{x^2}{2} \ln x - \frac{x^2}{4} (+c)$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>3</p>	<p>correct direction and sight of <math>\frac{1}{x}, \frac{x^2}{2}</math></p>
<p><b>(b)</b></p>	$y = (\ln x)^2$ $\left( \frac{dy}{dx} \right) = 2 \ln x \times \frac{1}{x}$	<p>M1</p> <p>A1</p>	<p>2</p>	<p><math>\frac{k}{x} \ln x</math> where <math>k = \frac{1}{2}, 1</math> or <math>2</math></p> <p><math>k = 2</math></p>
<p><b>(c)</b></p>	$y = \sqrt{x} \ln x$ $(V = \pi) \int_1^e x (\ln x)^2 \, dx$ $\left. \begin{aligned} u = (\ln x)^2 & \quad \frac{dv}{(dx)} = x \\ \frac{du}{(dx)} = 2 \ln x \frac{1}{x} & \quad v = \frac{x^2}{2} \end{aligned} \right\}$ $\int = \frac{x^2}{2} (\ln x)^2 - \int \frac{x^2}{2} \times \frac{2}{x} \ln x (dx)$ $= \frac{x^2}{2} (\ln x)^2 - \int x \ln x (dx)$ $= \frac{x^2}{2} (\ln x)^2 - \frac{1}{4} x^2 (2 \ln x - 1) \text{ OE}$ $V = (\pi) \left[ \frac{x^2}{2} (\ln x)^2 - \frac{1}{4} x^2 (2 \ln x - 1) \right]_1^e$ $= (\pi) \left[ \left( \frac{e^2}{2} - \frac{1}{4} e^2 \right) - \left( 0 + \frac{1}{4} \right) \right]$ $= \frac{\pi}{4} [e^2 - 1] \quad \text{OE}$	<p>B1</p> <p>M1</p> <p>m1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>6</p>	<p>all correct, incl brackets, <math>\pi</math>, limits and <math>dx</math> (but <math>dx</math> may be seen BEFORE this line)</p> <p>correct direction with <math>\frac{du}{(dx)} = \frac{k}{x} \ln x</math> where <math>k = \frac{1}{2}, 1</math> or <math>2</math> and sight of <math>\frac{x^2}{2}</math></p> <p>correct substitution of their terms into the parts formula</p> <p>integral needs to be simplified to <math>\int x \ln x</math></p> <p>correct substitution of 1 and e into their expressions of the form <math>px^2 (\ln x)^2 + qx^2 \ln x + rx^2</math> where <math>p, q</math> and <math>r</math> are non-zero rational numbers, and an intention to subtract</p> <p>Do not condone <math>F(1) - F(e)</math></p> <p><math>\pi \left[ \frac{e^2}{4} - \frac{1}{4} \right]</math> etc</p>
	<p><b>Total</b></p>		<p><b>11</b></p>	
	<p><b>TOTAL</b></p>		<p><b>75</b></p>	