General Certificate of Education
Advanced Subsidiary Examination
June 2013

Mathematics MPC1

Unit Pure Core 1

Monday 13 May 2013 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The line AB has equation $3x - 4y + 5 = 0$.

(a) The point with coordinates $(p, p+2)$ lies on the line AB. Find the value of the constant p. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The point A has coordinates $(1, 2)$. The point $C(-5, k)$ is such that AC is perpendicular to AB. Find the value of k. *(3 marks)*

(d) The line AB intersects the line with equation $2x - 5y = 6$ at the point D. Find the coordinates of D. *(3 marks)*

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2 (a) (i) Express $\sqrt{48}$ in the form $n\sqrt{3}$, where n is an integer. *(1 mark)*

(ii) Solve the equation

$$x\sqrt{12} = 7\sqrt{3} - \sqrt{48}$$

giving your answer in its simplest form. *(3 marks)*

(b) Express $\dfrac{11\sqrt{3} + 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}}$ in the form $m - \sqrt{15}$, where m is an integer. *(4 marks)*

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3 A circle C has equation

$$x^2 + y^2 - 10x + 14y + 25 = 0$$

(a) Write the equation of C in the form

$$(x - a)^2 + (y - b)^2 = k$$

where a, b and k are integers. *(3 marks)*

(b) Hence, for the circle C, write down:

(i) the coordinates of its centre; *(1 mark)*

(ii) its radius. *(1 mark)*

(c) (i) Sketch the circle C. *(2 marks)*

(ii) Write down the coordinates of the point on C that is furthest away from the x-axis. *(2 marks)*

(d) Given that k has the same value as in part **(a)**, describe geometrically the transformation which maps the circle with equation $(x+1)^2 + y^2 = k$ onto the circle C. *(3 marks)*

4 (a) The polynomial $f(x)$ is given by $f(x) = x^3 - 4x + 15$.

(i) Use the Factor Theorem to show that $x + 3$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x + 3)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(b) A curve has equation $y = x^4 - 8x^2 + 60x + 7$.

(i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Show that the x-coordinates of any stationary points of the curve satisfy the equation

$$x^3 - 4x + 15 = 0$$ *(1 mark)*

(iii) Use the results above to show that the only stationary point of the curve occurs when $x = -3$. *(2 marks)*

(iv) Find the value of $\dfrac{d^2y}{dx^2}$ when $x = -3$. *(3 marks)*

(v) Hence determine, with a reason, whether the curve has a maximum point or a minimum point when $x = -3$. *(1 mark)*

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5 (a) (i) Express $2x^2 + 6x + 5$ in the form $2(x + p)^2 + q$, where p and q are rational numbers. *(2 marks)*

(ii) Hence write down the minimum value of $2x^2 + 6x + 5$. *(1 mark)*

(b) The point A has coordinates $(-3, 5)$ and the point B has coordinates $(x, 3x + 9)$.

(i) Show that $AB^2 = 5(2x^2 + 6x + 5)$. *(3 marks)*

(ii) Use your result from part **(a)(ii)** to find the minimum value of the length AB as x varies, giving your answer in the form $\frac{1}{2}\sqrt{n}$, where n is an integer. *(2 marks)*

6 A curve has equation $y = x^5 - 2x^2 + 9$. The point P with coordinates $(-1, 6)$ lies on the curve.

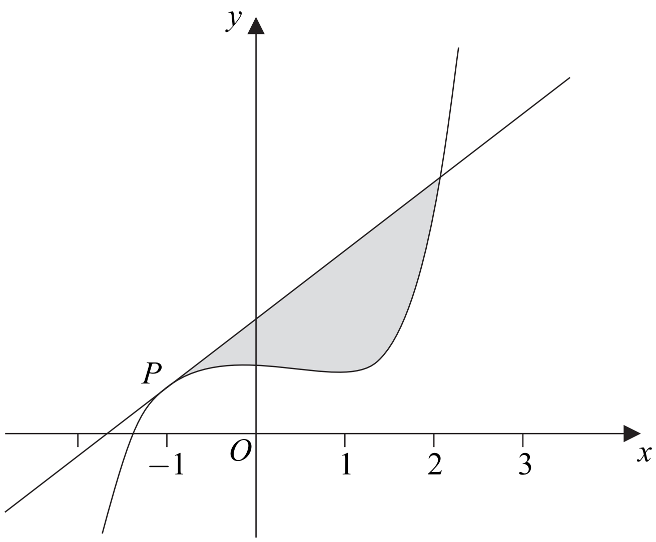
(a) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(5 marks)*

(b) The point Q with coordinates $(2, k)$ lies on the curve.

(i) Find the value of k. *(1 mark)*

(ii) Verify that Q also lies on the tangent to the curve at the point P. *(1 mark)*

(c) The curve and the tangent to the curve at P are sketched below.



(i) Find $\displaystyle\int_{-1}^{2} (x^5 - 2x^2 + 9)\,\mathrm{d}x$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the tangent to the curve at P. *(3 marks)*

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7 The quadratic equation

$$(2k - 7)x^2 - (k - 2)x + (k - 3) = 0$$

has real roots.

(a) Show that $7k^2 - 48k + 80 \leqslant 0$. *(4 marks)*

(b) Find the possible values of k. *(4 marks)*