



ADVANCED SUBSIDIARY GCE
MATHEMATICS
Further Pure Mathematics 1

4725

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

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1 Evaluate $\sum_{r=101}^{250} r^3$. [3]

2 The matrices \mathbf{A} and \mathbf{B} are given by $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$ and \mathbf{I} is the 2×2 identity matrix. Find the values of the constants a and b for which $a\mathbf{A} + b\mathbf{B} = \mathbf{I}$. [4]

3 The complex numbers z and w are given by $z = 5 - 2i$ and $w = 3 + 7i$. Giving your answers in the form $x + iy$ and showing clearly how you obtain them, find

(i) $4z - 3w$, [2]

(ii) z^*w . [2]

4 The roots of the quadratic equation $x^2 + x - 8 = 0$ are p and q . Find the value of $p + q + \frac{1}{p} + \frac{1}{q}$. [4]

5 The cubic equation $x^3 + 5x^2 + 7 = 0$ has roots α , β and γ .

(i) Use the substitution $x = \sqrt{u}$ to find a cubic equation in u with integer coefficients. [3]

(ii) Hence find the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. [2]

6 The complex number $3 - 3i$ is denoted by a .

(i) Find $|a|$ and $\arg a$. [2]

(ii) Sketch on a single Argand diagram the loci given by

(a) $|z - a| = 3\sqrt{2}$, [3]

(b) $\arg(z - a) = \frac{1}{4}\pi$. [3]

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z - a| \geq 3\sqrt{2} \quad \text{and} \quad 0 \leq \arg(z - a) \leq \frac{1}{4}\pi. \quad [3]$$

7 (i) Use the method of differences to show that

$$\sum_{r=1}^n \{(r+1)^4 - r^4\} = (n+1)^4 - 1. \quad [2]$$

(ii) Show that $(r+1)^4 - r^4 \equiv 4r^3 + 6r^2 + 4r + 1$. [2]

(iii) Hence show that

$$4 \sum_{r=1}^n r^3 = n^2(n+1)^2. \quad [6]$$

3

8 The matrix \mathbf{C} is given by $\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$.

(i) Draw a diagram showing the image of the unit square under the transformation represented by \mathbf{C} . [3]

The transformation represented by \mathbf{C} is equivalent to a transformation \mathbf{S} followed by another transformation \mathbf{T} .

(ii) Given that \mathbf{S} is a shear with the y -axis invariant in which the image of the point $(1, 1)$ is $(1, 2)$, write down the matrix that represents \mathbf{S} . [2]

(iii) Find the matrix that represents transformation \mathbf{T} and describe fully the transformation \mathbf{T} . [6]

9 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 2 \end{pmatrix}$.

(i) Find, in terms of a , the determinant of \mathbf{A} . [3]

(ii) Hence find the values of a for which \mathbf{A} is singular. [3]

(iii) State, giving a brief reason in each case, whether the simultaneous equations

$$ax + y + z = 2a,$$

$$x + ay + z = -1,$$

$$x + y + 2z = -1,$$

have any solutions when

(a) $a = 0$,

(b) $a = 1$.

[4]

10 The sequence u_1, u_2, u_3, \dots is defined by $u_1 = 3$ and $u_{n+1} = 3u_n - 2$.

(i) Find u_2 and u_3 and verify that $\frac{1}{2}(u_4 - 1) = 27$. [3]

(ii) Hence suggest an expression for u_n . [2]

(iii) Use induction to prove that your answer to part (ii) is correct. [5]



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