| | | | |
|---|---|---|---|
| **1** | First 2 terms in expansion = $1-5x$ | B1 | (simp to this, now or later) |
| | 3rd term shown as $\frac{-\frac{5}{3}.-\frac{8}{3}}{2}(3x)^2$ | M1 | $-\frac{8}{3}$ can be $-\frac{5}{3}-1$ |
| | | | $(3x)^2$ can be $9x^2$ or $3x^2$ |
| | $= +\ 20x^2$ | A1 | |
| | 4th term shown as $\frac{-\frac{5}{3}.-\frac{8}{3}.-\frac{11}{3}}{2.3}(3x)^3$ | M1 | $-\frac{11}{3}$ can be $-\frac{5}{3}-2$ |
| | | | $(3x)^3$ can be $27x^3$ or $3x^3$ |
| | $= -\frac{220}{3}x^3$ ISW | A1 | Accept $-\frac{440}{6}x^3$ ISW |

N.B. If 0, SR B2 to be awarded for $1-\frac{5}{3}x+\frac{20}{9}x^2-\frac{220}{81}x^3$. Do not mark $(1+x)^{-5/3}$ as a MR.

$\boxed{5}$

| | | | |
|---|---|---|---|
| **2** | Attempt quotient rule | M1 | |

[ Show fraction with denom $(1-\sin x)^2$ & num $+/-(1-\sin x)+/-\sin x+/-\cos x+/-\cos x$ ]

| | | | |
|---|---|---|---|
| | Numerator = $(1-\sin x).-\sin x-\cos x.-\cos x$ | A1 | terms in any order |

{ Product symbols must be clear or implied by further work }

| | | | |
|---|---|---|---|
| | Reduce correct numerator to $1-\sin x$ | B1 | or $-\sin x+\sin^2 x+\cos^2 x$ |
| | Simplify to $\frac{1}{1-\sin x}$ ISW | A1 | Accept $-\frac{1}{\sin x-1}$ |

$\boxed{4}$

| | | | |
|---|---|---|---|
| **3** | $\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}$ | M1 | For correct format |
| | $A(x-1)(x-2)+B(x-2)+C(x-1)^2 \equiv x^2$ | M1 | |
| | $A=-3$ | A1 | |
| | $B=-1$ | A1 | (B1 if cover-up rule used) |
| | $C=4$ | A1 | (B1 if cover-up rule used) |

[NB1: Partial fractions need not be written out; correct format + correct values sufficient.

NB2: Having obtained $B$ & $C$ by cover-up rule, candidates may substitute into general expression & algebraically manipulate; the M1 & A1 are then available if deserved.]

$\boxed{5}$

<u>These special cases using different formats are the only other ones to be considered</u> <u>Max</u>

| | | |
|---|---|---|
| $\frac{A}{x-1}+\frac{Bx+C}{(x-1)^2}+\frac{D}{x-2}$ ; | M1 M1; A0 for any values of $A$, $B$ & $C$, A1 or B1 for $D=4$ | 3 |
| $\frac{Ax+B}{(x-1)^2}+\frac{C}{x-2}$ ; | M0 M1; A1 for $A=-3$ <u>and</u> $B=2$, A1 or B1 for $C=4$ | 3 |

| | | | | |
|---|---|---|---|---|
| **4** | | Att by diff to connect d$x$ & d$u$ or find $\frac{dx}{du}$ or $\frac{du}{dx}$ (not d$x$=d<u>u</u>) | M1 | no accuracy; <u>not 'by parts'</u> |
| | | d$x$ = 2$u$ d$u$ or $\frac{du}{dx}=\frac{1}{2}(x+2)^{-\frac{1}{2}}$ AEF | A1 | |
| | | Indefinite integral $\rightarrow\int 2(u^2-2)^2\,(\frac{u}{u})(du)$ | A1 | May be implied later |
| | | {If relevant, cancel u/u and} attempt to square out | M1 | |
| | | {dep $\int kI(du)$ where $k=2$ or $\frac{1}{2}$ or 1 and I = $(u^2-2)^2$ or $(2-u^2)^2$ or $(u^2+2)^2$ } | | |
| | | Att to change limits if working with f($u$) after integration | M1 | or re-subst into integral attempt and use $-1$ & 7 |
| | | Indef integ = $\frac{2}{5}u^5+/-\frac{8}{3}u^3+8u$ or $\frac{1}{10}u^5+/-\frac{2}{3}u^3+2u$ | A1 | or $\frac{1}{5}u^5+/-\frac{4}{3}u^3+4u$ |
| | | $\frac{652}{15}$ or $43\frac{7}{15}$ ISW but no '+c' | A1 | |
| | | | $\boxed{7}$ | |
| **5** | | $\frac{d}{dx}(xy)=x\frac{dy}{dx}+y$ s.o.i. | B1 | Implied by e.g., $4x\frac{dy}{dx}+y$ |
| | | $\frac{d}{dx}(y^2)=2y\frac{dy}{dx}$ | B1 | |
| | | Diff eqn(=0 can be implied)(solve for $\frac{dy}{dx}$ and ) put $\frac{dy}{dx}=0$ | M1 | |
| | | Produce <u>only</u> $2x+4y=0$ (though AEF acceptable ) | *A1 | <u>without any error seen</u> |
| | | Eliminate $x$ or $y$ from curve eqn & eqn(s) just produced | M1 | |
| | | Produce either $x^2=36$ or $y^2=9$ | dep* A1 | Disregard other solutions |
| | | $\underline{(\pm6,\mp3)}$ AEF, as the only answer ISW | dep* A1 | Sign aspect must be clear |
| | | | $\boxed{7}$ | |
| **6** | **(i)** | State/imply scalar product of any two vectors = 0 | M1 | |
| | | Scalar product of correct two vectors = 4 + 2$a$ –6 | A1 | ( $4+2a-6=0\rightarrow$ M1A1) |
| | | $\underline{a=1}$ | A1 **3** | |
| | **(ii)** | **(a)** Attempt to produce at least two relevant equations | M1 | e.g. $2t=3+2s$ ….. |
| | | Solve two not containing '$a$' for $s$ and $t$ | M1 | |
| | | Obtain at least one of $s=-\frac{1}{2}$, $t=1$ | A1 | |
| | | Substitute in third equation & produce $\underline{a=-2}$ | A1 **4** | |
| | | **(b)** Method for finding magnitude of <u>any</u> vector | M1 | possibly involving '$a$' |
| | | Using $\cos\theta=\frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\boldsymbol{b}\rvert}$ for the pair of direction vectors | M1 | possibly involving '$a$' |
| | | <u>107, 108 (107.548) or 72, 73, 72.4, 72.5 (72.4516)</u> c.a.o. | A1 **3** | <u>1.87, 1.88 (1.87707) or 1.26</u> |
| | | | $\boxed{10}$ | |

| | | | | |
|---|---|---|---|---|
| **7** | **(i)** | Differentiate $x$ as a quotient, $\frac{v\,du - u\,dv}{v^2}$ or $\frac{u\,dv - v\,du}{v^2}$ | M1 | or product clearly defined |
| | | $\frac{dx}{dt} = -\frac{1}{(t+1)^2}$ or $\frac{-1}{(t+1)^2}$ or $-(t+1)^{-2}$ | A1 | WWW $\to$ 2 |
| | | $\frac{dy}{dt} = -\frac{2}{(t+3)^2}$ or $\frac{-2}{(t+3)^2}$ or $-2(t+3)^{-2}$ | B1 | |
| | | $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ | M1 | quoted/implied and used |
| | | $\frac{dy}{dx} = \frac{2(t+1)^2}{(t+3)^2}$ or $\frac{2(t+3)^{-2}}{(t+1)^{-2}}$ (dep 1st 4 marks) | *A1 | ignore ref $t=-1, t=-3$ |
| | | State <u>squares</u> +ve or $(t+1)^2$ & $(t+3)^2$ + ve $\therefore \frac{dy}{dx}$ +ve | dep*A1 **6** | or $\left(\frac{t+1}{t+3}\right)^2$ + ve . Ignore $\geq 0$ |
| | **(ii)** | Attempt to obtain $t$ from either the $x$ or $y$ equation | M1 | No accuracy required |
| | | $t = \frac{2-x}{x-1}$ AEF <u>or</u> $t = \frac{2}{y} - 3$ AEF | A1 | |
| | | Substitute in the equation not yet used in this part | M1 | or equate the 2 values of $t$ |
| | | Use correct meth to eliminate (‘double-decker’ ) fractions | M1 | |
| | | Obtain $\underline{2x + y = 2xy + 2}$ ISW AEF | A1 **5** | but not involving fractions $\boxed{11}$ |
| **8** | **(i)** | <u>Long division method</u> | | <u>Identity method</u> |
| | | Evidence of division process as far as 1st stage incl sub | M1 | $\equiv Q(x-1) + R$ |
| | | (Quotient = ) $x - 4$ | A1 | $Q = x - 4$ |
| | | (Remainder =) 2 ISW | A1 **3** | $R = 2$ ; N.B. might be B1 |
| | **(ii)** | **(a)** Separate variables; $\int \frac{1}{y-5}\,dy = \int \frac{x^2 - 5x + 6}{x-1}\,dx$ | M1 | ‘$\int$’ may be implied later |
| | | Change $\frac{x^2 - 5x + 6}{x-1}$ into their (Quotient + $\frac{\text{Rem}}{x-1}$ ) | M1 | |
| | | $\ln(y-5) = \sqrt{}$ (integration of their previous result) (+$c$)ISW | $\sqrt{}$A1 **3** | f.t. if using Quot + $\frac{\text{Rem}}{x-1}$ |
| | **(ii)** | **(b)** Substitute $y = 7$, $x = 8$ into their eqn containing ‘$c$’ | M1 | & attempt ‘$c$’ ( $-3.2$ , $\ln\frac{2}{49}$ ) |
| | | Substitute $x = 6$ and their value of ‘$c$’ | M1 | & attempt to find $y$ |
| | | <u>$y$ = 5.00 (5.002529)</u> Also $5 + \frac{50}{49}e^{-6}$ | A2 **4** | Accept 5, 5.0, |
| | | Beware: <u>any</u> wrong working anywhere $\to$ A0 even if answer is one of the acceptable ones. | | |
| | | | $\boxed{10}$ | |

| | | | |
|---|---|---|---|
| **9 (i)** | Attempt to multiply out $(x+\cos 2x)^2$ | M1 | Min of 2 correct terms |
| | <u>Finding $\int 2x\cos 2x\,dx$</u> | | |
| | Use $u = 2x, dv = \cos 2x$ | M1 | $1^{st}$ stage $f(x) +/- \int g(x)\,dx$ |
| | $1^{st}$ stage $x\sin 2x - \int \sin 2x\,dx$ | A1 | |
| | $\therefore \int 2x\cos 2x\,dx = x\sin 2x + \frac{1}{2}\cos 2x$ | A1 | |
| | <u>Finding $\int \cos^2 2x\,dx$</u> | | |
| | Change to $k\int +/-1+/-\cos 4x\,dx$ | M1 | where $k = \frac{1}{2}, 2$ or $1$ |
| | Correct version $\frac{1}{2}\int 1+\cos 4x\,dx$ | A1 | |
| | $\int \cos 4x\,dx = \frac{1}{4}\sin 4x$ | B1 | seen anywhere in this part |
| | Result = $\frac{1}{2}x + \frac{1}{8}\sin 4x$ | A1 | |
| | **(i)** ans $= \frac{1}{3}x^3 + x\sin 2x + \frac{1}{2}\cos 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x$ (+ c) | A1 **9** | Fully correct |
| **(ii)** | V = $\pi\int_0^{\frac{1}{2}\pi}(x+\cos 2x)^2\ (dx)$ | M1 | |
| | Use limits 0 & $\frac{1}{2}\pi$ correctly on their **(i)** answer | M1 | |
| | **(i)** correct value = $\frac{1}{24}\pi^3 - \frac{1}{2} + \frac{1}{4}\pi - \frac{1}{2}$ | A1 | |
| | Final answer = $\pi\left(\frac{1}{24}\pi^3 + \frac{1}{4}\pi - 1\right)$ | A1 **4** | **c.a.o.** No follow-through |
| | | $\boxed{\textbf{13}}$ | |

<u>Alternative methods</u>

**2** If $y = \dfrac{\cos x}{1-\sin x}$ is changed into $y(1-\sin x) = \cos x$, award

M1 for clear use of the product rule (though possibly trig differentiation inaccurate)

A1 for $-y\cos x + (1-\sin x)\frac{dy}{dx} = -\sin x$ AEF

B1 for reducing to a fraction with $1-\sin x$ or $-\sin x + \sin^2 x + \cos^2 x$ in the numerator

A1 for correct final answer of $\dfrac{1}{1-\sin x}$ or $(1-\sin x)^{-1}$

If $y = \dfrac{\cos x}{1-\sin x}$ is changed into $y = \cos x(1-\sin x)^{-1}$, award

M1 for clear use of the product rule (though possibly trig differentiation inaccurate)

A1 for $\left(\frac{dy}{dx}\right) = \cos^2 x(1-\sin x)^{-2} + (1-\sin x)^{-1}.-\sin x$ AEF

B1 for reducing to a fraction with $1-\sin x$ or $-\sin x+\sin^2 x+\cos^2 x$ in the numerator

A1 for correct final answer of $\frac{1}{1-\sin x}$ or $(1-\sin x)^{-1}$

**6(ii)(a)** If candidates use some long drawn-out method to find '*a*' instead of the direct route, allow

M1 as before, for producing the 3 equations

M1 for any satisfactory method which will/does produce 'a', however involved

A<u>2</u> for $a=-2$

**7(ii)** Marks for obtaining this Cartesian equation are not available in part (i).

<u>If part (ii) is done first and then part (i) is attempted using the Cartesian equation, award marks as follow:</u>

<u>Method 1 where candidates differentiate implicitly</u>

M1 for attempt at implicit differentiation

A1 for $\frac{dy}{dx}=\frac{2y-2}{1-2x}$ AEF

M1 for substituting parametric values of $x$ and $y$

A2 for simplifying to $\frac{2(t+1)^2}{(t+3)^2}$

A1 for finish as in original method

<u>Method 2 where candidates manipulate the Cartesian equation to find $x$ = or $y$=</u>

M1 for attempt to re-arrange so that either $y = \mathrm{f}(x)$ or $x = \mathrm{g}(y)$

A1 for correct $y=\frac{2-2x}{1-2x}$ AEF or $x=\frac{2-y}{2-2y}$ AEF

M1 for differentiating as a quotient

A2 for obtaining $\frac{dy}{dx}=\frac{2}{(1-2x)^2}$ or $\frac{(2-2y)^2}{2}$

A1 for finish as in original method

**8(ii)(b)** If definite integrals are used , then

M2 for $[\ ]_y^7=[\ ]_6^8$ or equivalent or M1 for $[\ ]_7^y=[\ ]_6^8$ or equivalent

A<u>2</u> for 5, 5.0, 5.00 (5.002529) <u>with caveat as in main scheme</u> dep M<u>2</u>