

GCE

Edexcel GCE

Mathematics

Statistics 4 (6686)

June 2008

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Mark Scheme (Final)

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Mathematics

June 2008  
6686 Statistics S4  
Mark Scheme

Question Number	Scheme	Marks
1 a	$E(\theta_1) = \frac{E(X_3) + E(X_4) + E(X_5)}{3}$ $= \frac{3\mu}{3}$ $= \mu \quad \text{Bias} = 0$ <p style="text-align: right;">allow unbiased</p> $E(\theta_2) = \frac{E(X_{10}) - E(X_1)}{3}$ $= 1/3(\mu - \mu)$ $= 0 \quad \text{Bias} = -\mu$ <p style="text-align: right;">allow <math>\pm \mu</math></p> $E(\theta_3) = \frac{3E(X_1) + 2E(X_2) + E(X_{10})}{6}$ $= \frac{3\mu + 2\mu + \mu}{6}$ $= \mu \quad \text{Bias} = 0$ <p style="text-align: right;">allow unbiased</p>	<p style="text-align: right;">B1</p> <p style="text-align: right;">B1,B1</p> <p style="text-align: right;">B1 (4)</p>
b	$\text{Var}(\theta_1) = \frac{1}{9} \{(\text{Var } X_2) + \text{Var}(X_3) + \text{Var}(X_4)\}$ $= \frac{1}{9} \{\sigma^2 + \sigma^2 + \sigma^2\}$ $= \frac{1}{3} \sigma^2$ $\text{Var}(\theta_2) = \frac{2}{9} \sigma^2$ $\text{Var}(\theta_3) = \frac{1}{36} \{9\sigma^2 + 4\sigma^2 + \sigma^2\}$ $= \frac{7}{18} \sigma^2$	<p style="text-align: right;">M1</p> <p style="text-align: right;">A1</p> <p style="text-align: right;">B1</p> <p style="text-align: right;">M1</p> <p style="text-align: right;">A1</p>
ci)	$\theta_1$ is the better estimator. It has a lower var. and no bias	<p style="text-align: right;">B1</p>
ii)	$\theta_2$ is the worst estimator. It is biased	<p style="text-align: right;">depB1 B1 depB1 (4)</p>

Question Number	Scheme	Marks
2 a	$H_1 : \sigma_A^2 = \sigma_B^2 \quad H_0 : \sigma_A^2 \neq \sigma_B^2$ $S_A^2 = 22.5 \quad s_B^2 = 21.6 \quad \text{awrt}$ $\frac{s_1^2}{s_2^2} = 1.04$ $F_{(8, 6)} = 4.15$ <p>1.04 &lt; 4.15 do not reject <math>H_0</math>. The variances are the same.</p>	<p>B1</p> <p>M1 A1A1</p> <p>M1 A1</p> <p>B1</p> <p>B1</p> <p>(8)</p>
b	<p>Assume the samples are selected at random, (independent)</p>	<p>B1</p> <p>(1)</p>
c	$s_p^2 = \frac{8(22.5) + 6(21.62)}{14} = 22.12 \quad \text{awrt 22.1}$ $H_0 : \mu_A = \mu_B \quad H_1 : \mu_A \neq \mu_B$ $t = \frac{40.667 - 39.57}{\sqrt{22.12} \sqrt{\frac{1}{9} + \frac{1}{7}}}$ $= 0.462 \quad 0.42 - 0.47$ <p>Critical value = <math>t_{14}(2.5\%) = 2.145</math></p> <p>0.462 &lt; 2.145 No evidence to reject <math>H_0</math>. The means are the same</p>	<p>M1 A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>(7)</p>
d	<p>Music has no effect on performance</p>	<p>B1</p> <p>(1)</p>

Question Number	Scheme	Marks
3	<p>Differences 2.1 -0.7 2.6 -1.7 3.3 1.6 1.7 1.2 1.6 2.4</p> <p><math>\bar{d} = 1.41</math></p> <p><math>H_0 : \mu_d = 0 \quad H_1 : \mu_d &gt; 0</math></p> <p><math>s = \sqrt{\frac{40.65 - 10 \times 1.41^2}{9}} = 1.5191\dots</math></p> <p><math>t = \frac{1.41}{\left(\frac{1.519\dots}{\sqrt{10}}\right)} = 2.935\dots</math>                      awrt 2.94 /2.93</p> <p><math>t_9 (1\%) = 2.821</math></p> <p>2.935.. &gt; 2.821 Evidence to reject <math>H_0</math>. There has been an increase in the mean weight of the mice.</p>	<p>M1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>M1 A1</p> <p>B1</p> <p>B1ft</p> <p>(8)</p>

2 sample test can score

M0 M0

B1 for  $H_0 : \mu_A = \mu_B \quad H_1 : \mu_A < \mu_B$

M1  $\frac{9 \times 24.5 + 9 \times 17.16}{18}$

M0 A0

B1 2.552

B1 ft

ie 4/8

Question Number	Scheme	Marks
4a	$\bar{x} = 668.125 \quad s = 84.428$ $T_7(5\%) = 1.895$ Confidence limits = $668.125 \pm \frac{1.895 \times 84.428}{\sqrt{8}}$ $= 611.6 \text{ and } 724.7$ Confidence interval = (612, 725)	M1 M1  B1  M1  A1A1  (6)
b	Normal distribution	B1 (1)
c	£650 is within the confidence interval. No need to worry.	B1 ✓ B1 ✓ (2)

Question Number	Scheme	Marks
5 a	Confidence interval = $\left( \frac{15 \times 0.003}{27.488}, \frac{15 \times 0.003}{6.262} \right)$ = (0.00164, 0.00719)	M1 B1B1 A1 A1 (5)
b	$0.07^2 = 0.0049$ 0.0049 is within the 95% confidence interval. There is no evidence to reject the idea that the standard deviation of the volumes is not 0.07 or The machine is working well.	M1 A1 A1 (3)

Question Number	Marks	Scheme										
6 a	$H_0 : p = 0.35$ $H_1 : p \neq 0.35$	B1 B1 (2)										
b	Let $X =$ Number cured then $X \sim B(20, 0.35)$  $\alpha = P(\text{Type I error}) = P(x \leq 3) + P(x \geq 11)$ given $p = 0.35$ = 0.0444 + 0.0532 = 0.0976	B1  M1  A1 (3)										
c	$\beta = P(\text{Type II error}) = P(4 \leq x \leq 10)$  <table style="margin-left: 20px;"> <tr> <td><math>p</math></td> <td>0.2</td> <td>0.3</td> <td>0.4</td> <td>0.5</td> </tr> <tr> <td><math>\beta</math></td> <td>0.5880</td> <td><b>0.8758</b></td> <td>0.8565</td> <td><b>0.5868</b></td> </tr> </table>	$p$	0.2	0.3	0.4	0.5	$\beta$	0.5880	<b>0.8758</b>	0.8565	<b>0.5868</b>	M1   A1A1 (3)
$p$	0.2	0.3	0.4	0.5								
$\beta$	0.5880	<b>0.8758</b>	0.8565	<b>0.5868</b>								
d	Power = $1 - B$ 0.4120   0.1435	M1 A1 (2)										
e	Not a good procedure. Better further away from 0.35 or This is not a very powerful test ( power = $1 - \beta$ )	B1 B1dep (2)										

Question Number	Scheme	Marks
7 a	<p><math>H_0 : \mu = 230 \quad H_1 : \mu &lt; 230</math></p> <p><math>\nu = 9</math></p> <p>From table critical value = <math>\pm 1.833</math></p> <p><math>\bar{x} = 228.3 \quad S = 17.858</math></p> $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$ $= \pm \frac{228.3 - 230}{\frac{17.858}{\sqrt{10}}} = \pm 0.301$ <p><math>\pm 0.301 &gt; \pm 1.833</math>. No evidence to reject <math>H_0</math>. Mean is 230 N/mm<sup>2</sup></p>	<p>B1</p> <p>B1√</p> <p>B1 B1</p> <p>M1</p> <p>A1</p> <p>B1</p>
b	<p>Since the tensile strength is the same and the price is cheaper recommend use new supplier.</p>	<p>(7)</p> <p>B1</p> <p>(1)</p>