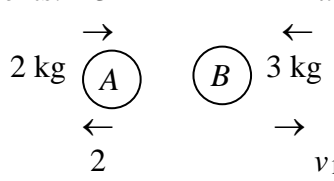
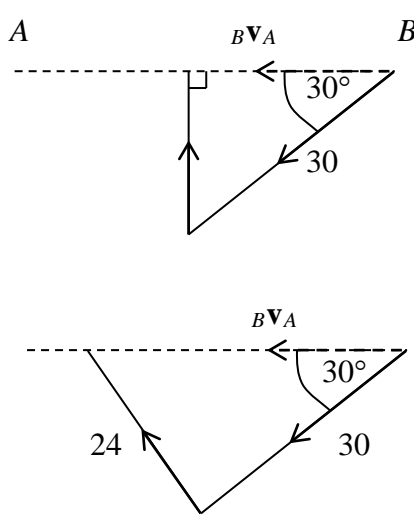
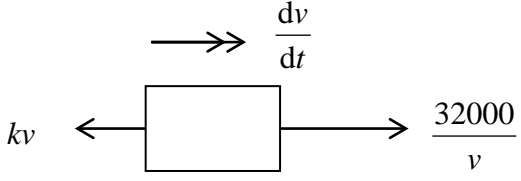


Question Number	Scheme	Marks
<p>1.</p>	<p><b>i</b> components: <math>3 \qquad u_1</math></p>  <p>Momentum <math>\leftarrow \rightarrow</math>: <math>2 \times 3 - 3u_1 = -2 \times 2 + 3v_1</math></p> <p>NLI: <math>10 = 3u_1 + 3v_1</math></p> <p><math>\frac{1}{2}(3 + u_1) = 2 + v_1</math></p> <p><math>1 = u_1 - 2v_1</math></p> <p>Solve for <math>u_1</math>, <math>23 = 9u_1 \Rightarrow u_1 = \frac{23}{9}</math></p> <p><b>j</b> component <math>= -u_1 \tan \alpha = -\frac{46}{9}</math></p> <p>Hence <math>\mathbf{u}_B = -\frac{23}{9} \mathbf{i} - \frac{46}{9} \mathbf{j}</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1 ft</p> <p>A1</p> <p><b>(9 marks)</b></p>
<p>2.</p>	 <p><math>Bv_A</math> as <math>\leftarrow</math></p> <p>Correct <math>\Delta</math> for <math>v_B</math> minimum</p> <p><math>v = 30 \sin 30^\circ = 15 \text{ km h}^{-1}</math></p> <p>Correct <math>\Delta</math></p> <p><math>\frac{30}{\sin \alpha} = \frac{24}{\sin 30^\circ}</math></p> <p><math>\Rightarrow \sin \alpha = \frac{5}{8}</math></p> <p><math> Bv_A  = 30 \cos 30^\circ + 24 \cos \alpha</math></p> <p><math>(\approx 44.716)</math></p> <p><math>T = \frac{20}{44.716} \approx 0.4473</math></p> <p><math>\Rightarrow 0927 \text{ hrs (awrt)}</math></p>	<p>M1</p> <p>M1</p> <p>A1 (3)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (7)</p> <p><b>(10 marks)</b></p>

(ft = follow through mark; awrt = anything which rounds to)

Question Number	Scheme	Marks
3. (a)	<div style="text-align: center;">  </div> $800 \frac{dv}{dt} = \frac{32000}{v} - kv$ $\Rightarrow 800v \frac{dv}{dt} = 32000 - kv^2 \quad (*)$	<p>M1 A1</p> <p>A1 (3)</p>
(b)	$v = 40, \quad \frac{dv}{dt} = 0 \Rightarrow 32000 = k \times 40^2$ $\Rightarrow k = 20$	<p>M1</p> <p>A1 (2)</p>
(c)	$\int dt = 800 \int \frac{v dv}{32000 - 20v^2} = \int \frac{40v dv}{1600 - v^2}$ $t = -20 \ln(1600 - v^2) (+ C)$ $t = 0, v = 0 \Rightarrow C = 20 \ln 1600 \text{ (or use of limits)}$ $t = 20 \ln 1600 - 20 \ln(1600 - v^2)$ $\Rightarrow t = 20 \ln \left( \frac{1600}{1600 - v^2} \right)$ $\frac{1600}{1600 - v^2} = e^{\frac{t}{20}}$ $1600 e^{-\frac{t}{20}} = 1600 - v^2$ $v = 40 \sqrt{\left( 1 - e^{-\frac{t}{20}} \right)}$	<p>M1</p> <p>M1 A1 ft</p> <p>M1 A1 ft</p> <p>M1</p> <p>A1 (7)</p> <p><b>(12 marks)</b></p>

((\*) indicates answer is given on the examination paper)

Question Number	Scheme	Marks
4.	(a) $AC = 4a \cos \theta \Rightarrow$ extension in spring $= 4a \cos \theta - 2a$	B1
	$V = -2mga \cos \theta - 2mg \times 3a \cos \theta + \frac{4mg}{4a}(4a \cos \theta - 2a) (+ C)$	M1 A1 A1
	$= -8mga \cos \theta + 4mg (2 \cos \theta - 1)^2 (+ C)$	
	$= 4mga[(2 \cos \theta - 1)^2 - 2 \cos \theta] (+ C)$	A1 (5)
	(b) $\frac{dV}{d\theta} = 4mga[2(2 \cos \theta - 1)(-2 \sin \theta) + 2 \sin \theta]$	M1 A1
$= 8mga \sin \theta (3 - 4 \cos \theta)$		
$= 0 \Rightarrow \cos \theta = \frac{3}{4} (\theta \neq 0, \pi)$	M1 A1	
$\Rightarrow \theta = 0.723$	A1 (4)	
(c) $\frac{d^2V}{d\theta^2} = 8mga \cos \theta (3 - 4 \cos \theta) + 32mga \sin^2 \theta$	M1 A1	
as $\theta = \frac{3}{4} \Rightarrow \frac{d^2V}{d\theta^2} = 0 + 32mga \times \frac{7}{16}$	M1	
$= 14mga$		
$> 0 \Rightarrow$ stable	A1 (4)	
	<b>(13 marks)</b>	

Question Number	Scheme	Marks
<p>5. (a)</p> <p>(b)</p>	$\mathbf{r}_P = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \quad \mathbf{r}_Q = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ <p><math>\Rightarrow t = 2</math> (one component)</p> <p>showing true for all components <math>\Rightarrow</math> collide</p> <p><math>\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}</math></p> $\mathbf{v}_R - \mathbf{v}_P = \lambda \begin{pmatrix} -5 \\ 4 \\ -1 \end{pmatrix}, \quad \mathbf{v}_R - \mathbf{v}_Q = \mu \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$ $\mathbf{v}_Q - \mathbf{v}_P = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \lambda \begin{pmatrix} -5 \\ 4 \\ -1 \end{pmatrix} - \mu \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \lambda \begin{pmatrix} -5 \\ 4 \\ -1 \end{pmatrix} - \mu \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$ $\left. \begin{aligned} -5\lambda + 2\mu &= 1 \\ 4\lambda - 2\mu &= -2 \\ -\lambda + \mu &= 2 \end{aligned} \right\}$ <p>Solve for either <math>\lambda = 1</math> or <math>\mu = 3</math></p> <p>Hence <math>\mathbf{v}_R = -4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}</math></p> $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + 2 \begin{pmatrix} -4 \\ 6 \\ -2 \end{pmatrix}$ <p><math>\Rightarrow a = 11, b = -10, c = 5</math></p> <p><math>t = 0, R</math> is at <math>11\mathbf{i} - 10\mathbf{j} + 5\mathbf{k}</math></p>	<p>B1 (either)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (5)</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>A1</p> <p>M1 A1ft</p> <p>A1 (9)</p> <p><b>(14 marks)</b></p>

(ft = follow through mark)

Question Number	Scheme	Marks
<p>6. (a)</p> <p>(b)</p>	<div style="text-align: center;"> </div> <p> <math>T_1 = 4(1 + x) ; T_2 = 4(1 + \frac{1}{2} \sin 4t - x)</math>  <math>T_2 - T_1 = 2\ddot{x}</math>  <math>2\ddot{x} = 4(1 + \frac{1}{2} \sin 4t - x) - 4(1 + x)</math>  <math>\Rightarrow \ddot{x} + 4x = \sin 4t \quad (*)</math> </p> <p>                     CF: <math>x = A \sin 2t + B \cos 2t</math>                      PI: <math>x = P \sin 4t</math>  <math>-16P \sin 4t + 4P \sin 4t = \sin 4t</math>  <math>\Rightarrow P = -\frac{1}{12}</math>  <math>x = A \sin 2t + B \cos 2t - \frac{1}{12} \sin 4t</math>  <math>t = 0, x = 0 \Rightarrow B = 0</math>  <math>\dot{x} = 2A \cos 2t - \frac{1}{3} \cos 4t</math>  <math>t = 0, \dot{x} = 0 \Rightarrow A = \frac{1}{6}</math>  <math>\dot{x} = 0 \Rightarrow \frac{1}{3} \cos 2t - \frac{1}{3} \cos 4t = 0</math>  <math>\Rightarrow \cos 4t = \cos 2t</math>  <math>\Rightarrow 4t = 2t + 2\pi \text{ or } 2\pi - 2t</math>  <math>\Rightarrow t = \pi \text{ or } \frac{\pi}{3}</math>  <math>\Rightarrow</math> First at rest after <math>t = 0</math> when <math>t = \frac{\pi}{3}</math> </p>	<p>B1; B1</p> <p>M1</p> <p>A1</p> <p>A1 (5)</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 ft (8)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1 cso (4)</p> <p><b>(17 marks)</b></p>

(ft = follow through mark; (\*) denotes answer is given on paper; cso = correct solution only)