A-LEVEL

Mathematics

Statistics 2B – MS2B

Mark scheme

6360
June 2014

Version/Stage: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

A-level Mathematics June 2014 MS2B

Q1	Solution	Marks	Total	Comments
(a)	Sample mean = $1904 \div 5 = 380.8$	B1		CAO
	$s = 4.38$ or $s^2 = 19.2$	B1		AWRT
	$t_4 = 2.132$	B1		AWRT 2.13
	C.I. = $380.8 \pm 2.132 \times \text{`}\frac{4.38}{\sqrt{5}}\text{'}$ or $\sqrt{(\text{`}19.2\text{'}/_5)}$	M1		Use of their $4.38/\sqrt{5}$ or $\sqrt{(\text{`}19.2\text{'}/_5)}$
		m1		Rest of formula (using t_4 or t_5 (2.015))
	= (377, 385)	A1		AWRT
			6	
(b)	3	B1		CAO
			1	
			7	

Q2	Solution	Marks	Total	Comments
(a)	<table><tr><th></th><th>E</th><th>S</th><th>W</th><th>NI</th><th>Total</th></tr><tr><td>Male</td><td>57</td><td>44</td><td>27</td><td>17</td><td>145</td></tr><tr><td>Female</td><td>39</td><td>43</td><td>19</td><td>4</td><td>105</td></tr><tr><td>Total</td><td>96</td><td>87</td><td>46</td><td>21</td><td>250</td></tr></table>	B2,1	**2**	B2 all correct, B1 one slip.
(b)	<table><tr><td>Expected</td><th>E</th><th>S</th><th>W</th><th>NI</th></tr><tr><td>Male</td><td>55.68</td><td>50.46</td><td>26.68</td><td>12.18</td></tr><tr><td>Female</td><td>40.32</td><td>36.54</td><td>19.32</td><td>8.82</td></tr></table> <table><tr><td>0.03129..</td><td>0.82702..</td><td>0.00383..</td><td>1.90742..</td></tr><tr><td>0.04321..</td><td>1.14207..</td><td>0.00530..</td><td>2.63405..</td></tr></table> Sum = 6.59 $\nu = (4 - 1)(2 - 1) = 3$ Critical value = 6.251 H_o:No association between country & gender H_1:Association between country & gender Test statistic in critical region, reject H_o There is significant evidence of association between country and gender.	M1 M1 A1 B1 B1 B1 A1 E1	**8**	Expected attempted, at least 2 correct to 3 s.f. $(O - E)^2/E$ attempted, at least 1 correct to 3 s.f. AWFW 6.58 – 6.60 CAO AWRT 6.25 B2 for just 6.25 seen At least 1 correct – must be in context. Comparison of 6.59 with 6.251 Dep on 6.59 A1 and 6.251 B1 and on hypotheses B1 Conclusion in context Dep on previous A1 and B1
(c)	More females than expected from Scotland Fewer females than expected from N.I. About the right number of females from England and/or Wales	B1	**1**	For any one of these
			11	

If they combine Wales and Northern Ireland

Q2	Solution	Marks	Total	Comments
(b)	<table><tr><td>Expected</td><th>E</th><th>S</th><th>W + NI</th></tr><tr><td>Male</td><td>55.68</td><td>50.46</td><td>38.86</td></tr><tr><td>Female</td><td>40.32</td><td>36.54</td><td>28.14</td></tr></table> <table><tr><td>0.03129..</td><td>0.82702..</td><td>0.67986..</td></tr><tr><td>0.04321..</td><td>1.14207..</td><td>0.93886</td></tr></table> Sum = 3.66 $\nu = (3 - 1)(2 - 1) = 2$ Critical value = 4.605 H_o:No association between country & gender H_1:Association between country & gender Test statistic not in critical region, accept H_o There is no significant evidence of association between country and gender.	M1 M1 A0 B1F B1F B1 A0 E0	A maximum of 5 out of 8	Expected attempted, at least 2 correct to 3 s.f. $(O - E)^2/E$ attempted, at least 1 correct to 3 s.f. AWFW 4.60 to 4.61 B2F for just 4.60 or 4.61 seen At least 1 correct – must be in context

Q3	Solution	Marks	Total	Comments
(a)	$P(X \leq 4) = 0.3$ So $P(\text{Both} \leq 4) = 0.3^2 = 0.09$	M1 A1	**2**	CAO
(b)(i)	$0.1 + 0.2 + a + 0.3 + b = 1$ so $a + b = 0.4$ $3 \times 0.1 + 4 \times 0.2 + 5a + 6 \times 0.3 + 7b = 5.1$ $5a + 7b = 2.2$ and $5a + 5b = 2.0$ or substitution of $b = 0.4 - a$ or $a = 0.4 - b$ leading to $a = 0.3$, $b = 0.1$	B1 M1 m1 A1	**4**	Correct treatment of simultaneous equations, starting with correctly simplified $5a + 7b = 2.2$ CAO
(ii)	$E(X^2) = 0.1\times3^2+0.2\times4^2+0.3\times5^2+0.3\times6^2+0.1\times7^2$ (= 27.3) $\text{Var}(X) = E(X^2) - E(X)^2 = 27.3 - 5.1^2 = 1.29$	M1 A1	**2**	Not simply $E(X^2) = 27.3$ AG
(iii)	Using $N = 2X - 5$ $E(N) = 2E(X) - 5 = 5.2$ $\text{Var}(N) = 2^2\text{Var}(X) = 5.16$ so $\sigma_N = 2.27$	M1 A1 A1	**3**	Or by use of 1, 3, 5, 7, 9 AWRT Or $2 \times \sqrt{1.29}$
			11	

Q4	Solution	Marks	Total	Comments
(a)(i)	Area of rectangle = 1 (or total probability) $= {}^1/_k \times (b - a) \rightarrow (b - a) = k$	M1 A1	**2**	AG
(ii)	$E(X) = {}^1/_2(a + b)$ (or $a + {}^1/_2k$)	B1	**1**	
(iii)	$E(X^2) = \int_a^b \frac{x^2}{k}\,dx$ $= [{}^1/_{3k}x^3]_a^b$ $= \frac{(b^3 - a^3)}{3(b - a)} = {}^1/_3(b^2 + ab + a^2)$	M1 m1 A1	**3**	k or $(b - a)$ For integration. Ignore limits Use of correct limits AG
(iv)	$\text{Var}(X) = E(X^2) - [E(X)]^2$ $= {}^4/_{12}(b^2 + ab + a^2) - {}^3/_{12}(a + b)^2$ $= {}^1/_{12}(b^2 - 2ab + a^2) = {}^1/_{12}(b - a)^2$.	M1 A1	**2**	Applied to this case (their mean) Either form or continued to ${}^1/_{12}k^2$
(b)	${}^1/_{12}(b - a)^2 = 3 \rightarrow (b - a) = 6$ b = 10 $E(X) = {}^1/_2(a + b) = 7$	M1 A1 A1	**3**	
			11	

Q5	Solution	Marks	Total	Comments
(a)	$\mu = 128 \div 40 = 3.2$ as required for λ $s^2 = 3.2410\ldots$ (Condone $\sigma^2 = 3.16$) which is close to λ, as required for Poisson	B1 B1 E1	**3**	 AWRT 3.24 or 3.16 Clearly stated (for either s^2 or σ^2)
(b)(i)	$1 - P(X \leq 5) = 1 - 0.8946$ $= 0.105(4)$	M1 A1	**2**	For attempt to subtract $P(X \leq 5)$ AWRT
(ii)	$P(X \leq 7) - P(X \leq 2)$ $0.9832 - 0.3799$ $= 0.603(3)$	M1 B1 A1	**3**	Attempt to use these two For either. AWFW 0.603 to 0.604
(iii)	$P(X = 0) = 0.0408$ or $e^{-3.2}$ or $P(X \geq 0) = 0.9592$ $1 - 0.9592^2$ (or $0.0408^2 + 2\times 0.0408 \times 0.9592$) $= 0.0799$	 B1 M1 A1	**3**	 For any of these seen to 3 d.p. AWFW 0.079 to 0.081
(c)	Using Po(8.2) $e^{-8.2} \times 8.2^9 \div 9! + e^{-8.2} \times 8.2^{10} \div 10!$ $= 0.231$	M1 m1 A1	**3**	Stated or use in formula or either of figures below seen Or Calc $P(\leq 10) - P(\leq 8)$ $= 0.79555 - 0.56465$ AWRT
			14	

Q6	Solution	Marks	Total	Comments
(a)	$H_0: \mu = 20$, $H_1: \mu \neq 20$ $\bar{x} = 22.625$ $s = 4.5650066$ (or $\sigma = 4.27$) test stat = $\frac{22.625 - 20}{(4.5650066 \div \sqrt{8})}$ $= 1.626$ $t_7 = \pm 1.895$ Test statistic not in critical region, accept H_o There is insufficient evidence that Gary does not take a mean time of 20 minutes for an annual service. Alternative: If the boundaries of the critical region are calculated, marks as above except $20 \pm 1.895 \times (4.5650066 \div \sqrt{8})$ M1 ((16.94), 23.06) A1 (AWRT)	B1 B1 B1 M1 A1 B1 A1 E1	**8**	Both CAO AWFW 4.56 – 4.57 (or AWRT 4.27) Or $\sqrt{7}$ if $\sigma = 4.27$ used AWRT 1.63 Comparison of test stat with t_7 In context. These last two marks dep on both A1s and hypotheses B1. E1 also dep on previous A1.
(b)	5% sig gives $z = 1.64$ to 1.65 $20 + 1.6449 \times (4.6 \div \sqrt{100})$ $= 20 + 0.754$ to 0.759 So to not support suspicion need $\bar{x} \leq 20.75$ SC 20.76 using this method scores B1, M1, A1, A0	B1 M1 A1 A1	**4**	AWFW OE AWFW
			12	

Q7	Solution	Marks	Total	Comments
(a)	$P(X < 1) = \int_0^1 \frac{4x}{5}\,dx$ or $\frac{1}{2} \times 1 \times \frac{4}{5}$ $= \left[\frac{2}{5}x^2\right]_0^1 = \frac{2}{5}$	M1 A1	**2**	Including limits
(b)(i)	$\int_1^x \frac{1}{20}(3t^2 - 20t + 33)\,dt$ $= \left[\frac{1}{20}(t^3 - 10t^2 + 33t)\right]_1^x$ $= \frac{1}{20}(x^3 - 10x^2 + 33x) - \frac{1}{20}(1 - 10 + 33)$ $F(x) = \frac{2}{5} + \frac{1}{20}(x^3 - 10x^2 + 33x) - \frac{24}{20}$ $= \frac{1}{20}(x^3 - 10x^2 + 33x - 16)$	M1 A1 m1 A1	**4**	Accept x integral Correct integration with limits Use of limits With $\frac{2}{5}$ included AG
(ii)	$F(1.13) = 0.49819\ldots$ $F(1.14) = 0.50527\ldots$ Median requires $F(x) = 0.5$ $0.49819\ldots < 0.5 < 0.50527\ldots$ So $1.13 <$ median < 1.14 Alternative scheme for **(b)(ii)** If a calculator, or trial and improvement, has been used to solve the cubic equation directly: $\frac{1}{20}(x^3 - 10x^2 + 33x - 16) = 0.5$ median = AWFW 1.132 to 1.133 which lies between 1.13 and 1.14	B1 B1 E1 M1 A1 E1	**3**	At least 3 s.f. At least 3 s.f. Must clearly indicate that median requires $F(x) = 0.5$
			9	