

Mark Scheme 4725 June 2007

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1	<p><i>EITHER</i> $a = 2$</p> <p>$b = 2\sqrt{3},$ <i>OR</i></p> <p>$a = 2 \quad b = 2\sqrt{3}$</p>	<p>M1 A1 M1 A1 M1 M1 A1 A1</p>	<p>4</p> <p>4</p>	<p>Use trig to find an expression for a (or b) Obtain correct answer Attempt to find other value Obtain correct answer a.e.f. (Allow 3.46) State 2 equations for a and b</p> <p>Attempt to solve these equations Obtain correct answers a.e.f. SR \pm scores A1 only</p>
2	<p>$(1^3 =) \frac{1}{4} \times 1^2 \times 2^2$</p> <p>$\frac{1}{4} n^2(n+1)^2 + (n+1)^3$</p> <p>$\frac{1}{4} (n+1)^2(n+2)^2$</p>	<p>B1 M1 M1(indep) A1 A1</p>	<p>5</p> <p>5</p>	<p>Show result true for $n = 1$</p> <p>Add next term to given sum formula Attempt to factorise and simplify Correct expression obtained convincingly</p> <p>Specific statement of induction conclusion</p>
3	<p>$3\Sigma r^2 - 3\Sigma r + \Sigma 1$</p> <p>$3\Sigma r^2 = \frac{1}{2} n(n+1)(2n+1)$</p> <p>$3\Sigma r = \frac{3}{2} n(n+1)$</p> <p>$\Sigma 1 = n$ n^3</p>	<p>M1 A1 A1 A1 M1 A1</p>	<p>6</p> <p>6</p>	<p>Consider the sum of three separate terms</p> <p>Correct formula stated</p> <p>Correct formula stated</p> <p>Correct term seen Attempt to simplify Obtain given answer correctly</p>
4	<p>(i) $\frac{1}{2} \begin{pmatrix} 5 & -1 \\ -3 & 1 \end{pmatrix}$</p> <p>(ii)</p> <p>$\frac{1}{2} \begin{pmatrix} 2 & 0 \\ 23 & -5 \end{pmatrix}$</p>	<p>B1 B1 M1 M1(indep) A1ft A1ft</p>	<p>2</p> <p>4</p> <p>6</p>	<p>Transpose leading diagonal and negate other diagonal or solve sim. eqns. to get 1st column Divide by the determinant or solve 2nd pair to get 2nd column</p> <p>Attempt to use $B^{-1}A^{-1}$ or find B Attempt at matrix multiplication One element correct, a.e.f. All elements correct, a.e.f. NB ft consistent with their (i)</p>

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5	<p>(i) $\frac{1}{r(r+1)}$</p> <p>(ii) $1 - \frac{1}{n+1}$</p> <p>(iii) $S_{\infty} = 1$</p> $\frac{1}{n+1}$	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1ft</p> <p>M1</p> <p>A1 c.a.o.</p>	<p>1</p> <p>3</p> <p>3</p> <p>7</p>	<p>Show correct process to obtain given result</p> <p>Express terms as differences using (i)</p> <p>Show that terms cancel</p> <p>Obtain correct answer, must be n not any other letter</p> <p>State correct value of sum to infinity</p> <p>Ft their (ii)</p> <p>Use sum to infinity – their (ii)</p> <p>Obtain correct answer a.e.f.</p>
6	<p>(i) (a) $\alpha + \beta + \gamma = 3, \alpha\beta + \beta\gamma + \gamma\alpha = 2$</p> <p>(b)</p> $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ $= 9 - 4 = 5$ $\frac{3}{u^3} - \frac{9}{u^2} + \frac{6}{u} + 2 = 0$ <p>(ii) (a) $2u^3 + 6u^2 - 9u + 3 = 0$</p> $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -3$ <p>(b)</p>	<p>B1 B1</p> <p>M1</p> <p>A1 ft</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1ft</p>	<p>2</p> <p>2</p> <p>2</p> <p>2</p> <p>8</p>	<p>State correct values</p> <p>State or imply the result and use their values</p> <p>Obtain correct answer</p> <p>Use given substitution to obtain an equation</p> <p>Obtain correct answer</p> <p>Required expression is related to new cubic stated or implied</p> <p>-(their “b” / their “a”)</p>

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7	<p>(i)</p> $a(a - 12) + 32$ <p>(ii)</p> $\det \mathbf{M} = 12$ <p>non-singular</p> <p>(iii) <i>EITHER</i></p> <p><i>OR</i></p>	<p>M1 M1 A1</p> <p>3</p> <p>M1 A1ft B1</p> <p>2</p> <p>M1 A1</p> <p>3</p> <p>M1 A1 A1</p> <p>8</p>	<p>Show correct expansion process Show evaluation of a 2 x 2 determinant Obtain correct answer a.e.f.</p> <p>Substitute $a = 2$ in their determinant</p> <p>Obtain correct answer and state a consistent conclusion</p> <p>det $M = 0$ so non-unique solutions</p> <p>Attempt to solve and obtain 2 inconsistent equations Deduce that there are no solutions</p> <p>Substitute $a = 4$ and attempt to solve Obtain 2 correct inconsistent equations Deduce no solutions</p>
8	<p>(i) Circle, centre (3, 0), y-axis a tangent at origin Straight line, through (1, 0) with +ve slope In 1st quadrant only</p> <p>(ii) Inside circle, below line, above x-axis</p>	<p>B1B1 B1 B1 B1 B1 B2ft</p> <p>6 2 8</p>	<p>Sketch showing correct features N.B. treat 2 diagrams asa MR</p> <p>Sketch showing correct region SR: B1ft for any 2 correct features</p>

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9	<p>(i) $\begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$</p> <p>(ii) Rotation (centre O), 45°, clockwise</p> <p>(iii)</p> <p>(iv) $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix}$</p> <p>(v) $\det \mathbf{C} = 2$ area of square has been doubled</p>	<p>B1</p> <p>B1B1B1</p> <p>B1</p> <p>M1 A1</p> <p>B1</p> <p>B1</p>	<p>1</p> <p>3</p> <p>1</p> <p>2</p> <p>2</p> <p>2</p> <p>9</p>	<p>Correct matrix</p> <p>Sensible alternatives OK, must be a single transformation</p> <p>Matrix multiplication or combination of transformations</p> <p>For at least two correct images For correct diagram</p> <p>State correct value</p> <p>State correct relation a.e.f.</p>
10	<p>(i)</p> <p>$x^2 - y^2 = 16$ and $xy = 15$</p> <p>$\pm(5 + 3i)$</p> <p>(ii)</p> <p>$z = 1 \pm \sqrt{16 + 30i}$</p> <p>$6 + 3i, -4 - 3i$</p>	<p>M1</p> <p>A1A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1*</p> <p>A1 *M1dep A1 A1ft</p>	<p>6</p> <p>5</p> <p>11</p>	<p>Attempt to equate real and imaginary parts of $(x + iy)^2$ and $16 + 30i$</p> <p>Obtain each result</p> <p>Eliminate to obtain a quadratic in x^2 or y^2</p> <p>Solve to obtain $x = (\pm) 5$ or $y = (\pm) 3$</p> <p>Obtain correct answers as complex numbers</p> <p>Use quadratic formula or complete the square</p> <p>Simplify to this stage Use answers from (i) Obtain correct answers</p>

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