

Centre Number					Candidate Number				
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For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
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5	
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7	
TOTAL	



General Certificate of Education
Advanced Level Examination
June 2010

Mathematics

MM03

Unit Mechanics 3

Tuesday 22 June 2010 1.30 pm to 3.00 pm

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.



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Answer **all** questions in the spaces provided.

1 A tank containing a liquid has a small hole in the bottom through which the liquid escapes. The speed, $u \text{ m s}^{-1}$, at which the liquid escapes is given by

$$u = CV\rho g$$

where $V \text{ m}^3$ is the volume of the liquid in the tank, $\rho \text{ kg m}^{-3}$ is the density of the liquid, g is the acceleration due to gravity and C is a constant.

By using dimensional analysis, find the dimensions of C . (5 marks)

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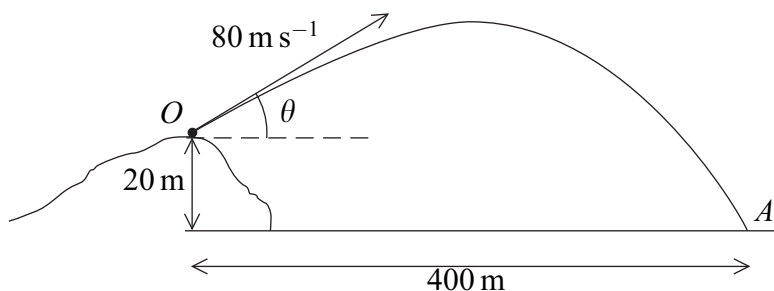


2 A projectile is fired from a point O on top of a hill with initial velocity 80 m s^{-1} at an angle θ above the horizontal and moves in a vertical plane. The horizontal and upward vertical distances of the projectile from O are x metres and y metres respectively.

(a) (i) Show that, during the flight, the equation of the trajectory of the projectile is given by

$$y = x \tan \theta - \frac{gx^2}{12800} (1 + \tan^2 \theta) \quad (5 \text{ marks})$$

(ii) The projectile hits a target A , which is 20 m vertically below O and 400 m horizontally from O .



Show that

$$49 \tan^2 \theta - 160 \tan \theta + 41 = 0 \quad (2 \text{ marks})$$

(b) (i) Find the two possible values of θ . Give your answers to the nearest 0.1° . (3 marks)

(ii) Hence find the shortest possible time of the flight of the projectile from O to A . (2 marks)

(c) State a necessary modelling assumption for answering part **(a)(i)**. (1 mark)

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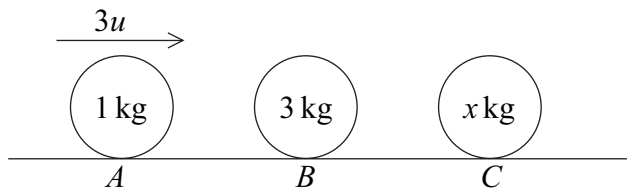


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3 Three smooth spheres, A , B and C , of equal radii have masses 1 kg , 3 kg and $x \text{ kg}$ respectively. The spheres lie at rest in a straight line on a smooth horizontal surface with B between A and C . The sphere A is projected with speed $3u$ directly towards B and collides with it.



The coefficient of restitution between each pair of spheres is $\frac{1}{3}$.

(a) Show that A is brought to rest by the impact and find the speed of B immediately after the collision in terms of u . (6 marks)

(b) Subsequently, B collides with C .

Show that the speed of C immediately after the collision is $\frac{4u}{3+x}$.

Find the speed of B immediately after the collision in terms of u and x . (6 marks)

(c) Show that B will collide with A again if $x > 9$. (2 marks)

(d) Given that $x = 5$, find the magnitude of the impulse exerted on C by B in terms of u . (2 marks)

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- 4** The unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k} are directed east, north and vertically upwards respectively.
- At time $t = 0$, the position vectors of two small aeroplanes, A and B , relative to a fixed origin O are $(-60\mathbf{i} + 30\mathbf{k})$ km and $(-40\mathbf{i} + 10\mathbf{j} - 10\mathbf{k})$ km respectively.
- The aeroplane A is flying with constant velocity $(250\mathbf{i} + 50\mathbf{j} - 100\mathbf{k})$ km h⁻¹ and the aeroplane B is flying with constant velocity $(200\mathbf{i} + 25\mathbf{j} + 50\mathbf{k})$ km h⁻¹.
- (a)** Write down the position vectors of A and B at time t hours. (3 marks)
 - (b)** Show that the position vector of A relative to B at time t hours is $((-20 + 50t)\mathbf{i} + (-10 + 25t)\mathbf{j} + (40 - 150t)\mathbf{k})$ km. (2 marks)
 - (c)** Show that A and B do not collide. (4 marks)
 - (d)** Find the value of t when A and B are closest together. (6 marks)

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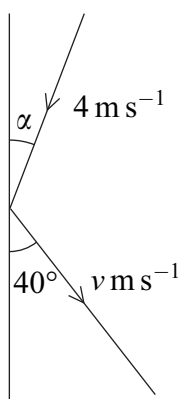


5 A smooth sphere is moving on a smooth horizontal surface when it strikes a smooth vertical wall and rebounds.

Immediately before the impact, the sphere is moving with speed 4 m s^{-1} and the angle between the sphere's direction of motion and the wall is α .

Immediately after the impact, the sphere is moving with speed $v\text{ m s}^{-1}$ and the angle between the sphere's direction of motion and the wall is 40° .

The coefficient of restitution between the sphere and the wall is $\frac{2}{3}$.



- (a)** Show that $\tan \alpha = \frac{3}{2} \tan 40^\circ$. (3 marks)
- (b)** Find the value of v . (3 marks)

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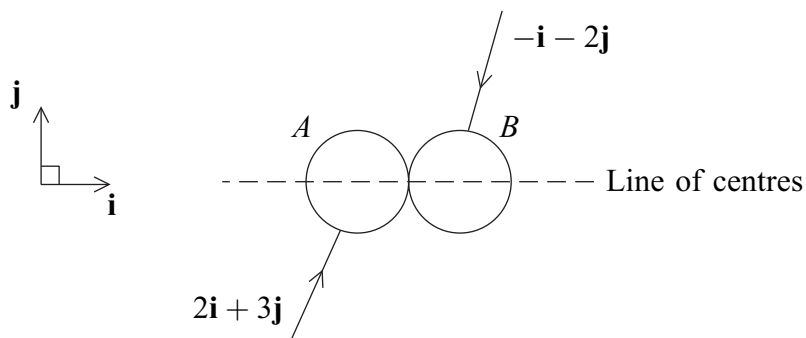


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6 Two smooth spheres, A and B , have equal radii and masses 1 kg and 2 kg respectively.

The sphere A is moving with velocity $(2\mathbf{i} + 3\mathbf{j}) \text{ m s}^{-1}$ and the sphere B is moving with velocity $(-\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$ on the same smooth horizontal surface.

The spheres collide when their line of centres is parallel to the unit vector \mathbf{i} , as shown in the diagram.



(a) Briefly state why the components of the velocities of A and B parallel to the unit vector \mathbf{j} are not changed by the collision. *(1 mark)*

(b) The coefficient of restitution between the spheres is 0.5.

Find the velocities of A and B immediately after the collision. *(6 marks)*

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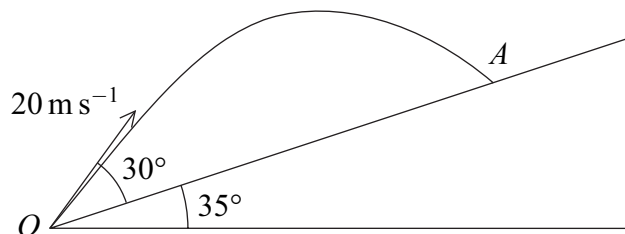
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7 A ball is projected from a point O on a smooth plane which is inclined at an angle of 35° above the horizontal. The ball is projected with velocity 20 m s^{-1} at an angle of 30° above the plane, as shown in the diagram. The motion of the ball is in a vertical plane containing a line of greatest slope of the inclined plane. The ball strikes the inclined plane at the point A .



- (a) Find the components of the velocity of the ball, parallel and perpendicular to the plane, as it strikes the inclined plane at A . (7 marks)
- (b) On striking the plane at A , the ball rebounds. The coefficient of restitution between the plane and the ball is $\frac{4}{5}$.

Show that the ball next strikes the plane at a point lower down than A . (6 marks)

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END OF QUESTIONS

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