



**ADVANCED GCE**  
**MATHEMATICS**  
Further Pure Mathematics 2

**4726**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

None

**Friday 22 May 2009**  
**Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

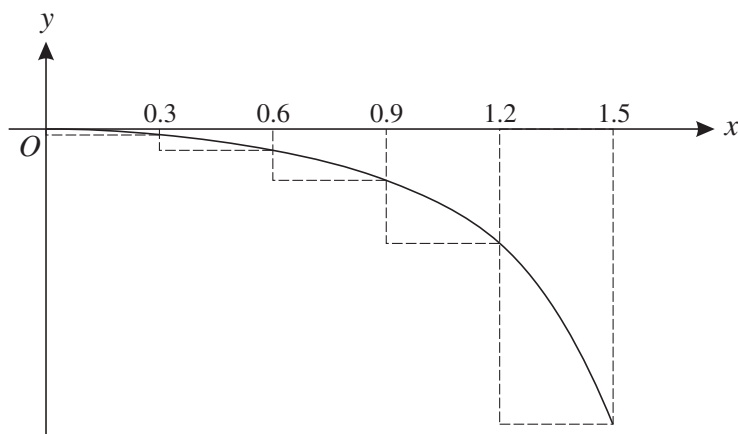
- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

## 2

1



The diagram shows the curve with equation  $y = \ln(\cos x)$ , for  $0 \leq x \leq 1.5$ . The region bounded by the curve, the  $x$ -axis and the line  $x = 1.5$  has area  $A$ . The region is divided into five strips, each of width 0.3.

(i) By considering the set of rectangles indicated in the diagram, find an upper bound for  $A$ . Give the answer correct to 3 decimal places. [2]

(ii) By considering another set of five suitable rectangles, find a lower bound for  $A$ . Give the answer correct to 3 decimal places. [2]

(iii) How could you reduce the difference between the upper and lower bounds for  $A$ ? [1]

2 Given that  $y = \frac{x^2 + x + 1}{(x - 1)^2}$ , prove that  $y \geq \frac{1}{4}$  for all  $x \neq 1$ . [4]

3 (i) Given that  $f(x) = e^{\sin x}$ , find  $f'(0)$  and  $f''(0)$ . [4]

(ii) Hence find the first three terms of the Maclaurin series for  $f(x)$ . [2]

4 Express  $\frac{x^3}{(x - 2)(x^2 + 4)}$  in partial fractions. [6]

5 It is given that  $I = \int_0^{\frac{1}{2}\pi} \frac{\cos \theta}{1 + \cos \theta} d\theta$ .

(i) By using the substitution  $t = \tan \frac{1}{2}\theta$ , show that  $I = \int_0^1 \left( \frac{2}{1+t^2} - 1 \right) dt$ . [5]

(ii) Hence find  $I$  in terms of  $\pi$ . [2]

3

6 Given that

$$\int_0^1 \frac{1}{\sqrt{16+9x^2}} dx + \int_0^2 \frac{1}{\sqrt{9+4x^2}} dx = \ln a,$$

find the exact value of  $a$ .

[6]

7 (i) Sketch the graph of  $y = \coth x$ , and give the equations of any asymptotes.

[3]

(ii) It is given that  $f(x) = x \tanh x - 2$ . Use the Newton-Raphson method, with a first approximation  $x_1 = 2$ , to find the next three approximations  $x_2$ ,  $x_3$  and  $x_4$  to a root of  $f(x) = 0$ . Give the answers correct to 4 decimal places.

[4]

(iii) If  $f(x) = 0$ , show that  $\coth x = \frac{1}{2}x$ . Hence write down the roots of  $f(x) = 0$ , correct to 4 decimal places.

[3]

8 (i) Using the definitions of  $\sinh x$  and  $\cosh x$  in terms of  $e^x$  and  $e^{-x}$ , show that

(a)  $\cosh(\ln a) \equiv \frac{a^2 + 1}{2a}$ , where  $a > 0$ ,

[3]

(b)  $\cosh x \cosh y - \sinh x \sinh y \equiv \cosh(x - y)$ .

[3]

(ii) Use part (i)(b) to show that  $\cosh^2 x - \sinh^2 x \equiv 1$ .

[1]

(iii) Given that  $R > 0$  and  $a > 1$ , find  $R$  and  $a$  such that

$$13 \cosh x - 5 \sinh x \equiv R \cosh(x - \ln a).$$

[5]

(iv) Hence write down the coordinates of the minimum point on the curve with equation  $y = 13 \cosh x - 5 \sinh x$ .

[2]

9 (i) It is given that, for non-negative integers  $n$ ,

$$I_n = \int_0^{\frac{1}{2}\pi} \sin^n \theta d\theta.$$

Show that, for  $n \geq 2$ ,

$$nI_n = (n - 1)I_{n-2}.$$

[4]

(ii) The equation of a curve, in polar coordinates, is

$$r = \sin^3 \theta, \quad \text{for } 0 \leq \theta \leq \pi.$$

(a) Find the equations of the tangents at the pole and sketch the curve.

[4]

(b) Find the exact area of the region enclosed by the curve.

[6]

There are no questions printed on this page.

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