OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS **4726**

Further Pure Mathematics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 22 May 2009
Morning

Duration: 1 hour 30 minutes

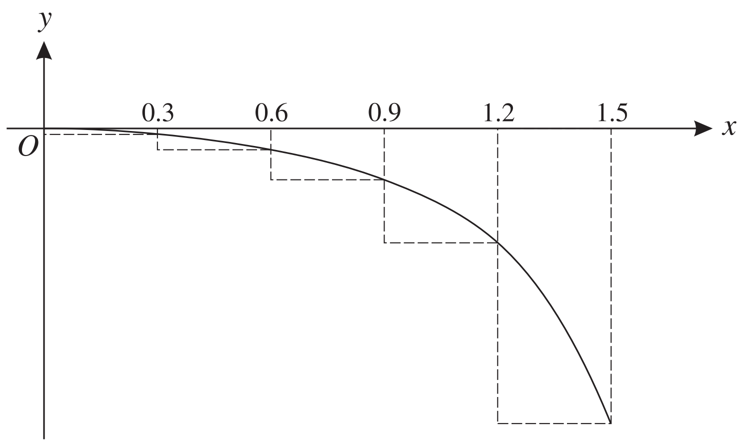
INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1



The diagram shows the curve with equation $y = \ln(\cos x)$, for $0 \leqslant x \leqslant 1.5$. The region bounded by the curve, the x-axis and the line $x = 1.5$ has area A. The region is divided into five strips, each of width 0.3.

(i) By considering the set of rectangles indicated in the diagram, find an upper bound for A. Give the answer correct to 3 decimal places. **[2]**

(ii) By considering another set of five suitable rectangles, find a lower bound for A. Give the answer correct to 3 decimal places. **[2]**

(iii) How could you reduce the difference between the upper and lower bounds for A? **[1]**

2 Given that $y = \dfrac{x^2 + x + 1}{(x-1)^2}$, prove that $y \geqslant \frac{1}{4}$ for all $x \neq 1$. **[4]**

3 **(i)** Given that $\mathrm{f}(x) = \mathrm{e}^{\sin x}$, find $\mathrm{f}'(0)$ and $\mathrm{f}''(0)$. **[4]**

(ii) Hence find the first three terms of the Maclaurin series for $\mathrm{f}(x)$. **[2]**

4 Express $\dfrac{x^3}{(x-2)(x^2+4)}$ in partial fractions. **[6]**

5 It is given that $I = \displaystyle\int_0^{\frac{1}{2}\pi} \frac{\cos\theta}{1+\cos\theta}\,\mathrm{d}\theta$.

(i) By using the substitution $t = \tan\frac{1}{2}\theta$, show that $I = \displaystyle\int_0^1 \left(\frac{2}{1+t^2} - 1\right)\mathrm{d}t$. **[5]**

(ii) Hence find I in terms of π. **[2]**

6 Given that

$$\int_0^1 \frac{1}{\sqrt{16+9x^2}}\,dx + \int_0^2 \frac{1}{\sqrt{9+4x^2}}\,dx = \ln a,$$

find the exact value of a. **[6]**

7 **(i)** Sketch the graph of $y = \coth x$, and give the equations of any asymptotes. **[3]**

(ii) It is given that $f(x) = x\tanh x - 2$. Use the Newton-Raphson method, with a first approximation $x_1 = 2$, to find the next three approximations x_2, x_3 and x_4 to a root of $f(x) = 0$. Give the answers correct to 4 decimal places. **[4]**

(iii) If $f(x) = 0$, show that $\coth x = \frac{1}{2}x$. Hence write down the roots of $f(x) = 0$, correct to 4 decimal places. **[3]**

8 **(i)** Using the definitions of $\sinh x$ and $\cosh x$ in terms of e^x and e^{-x}, show that

(a) $\cosh(\ln a) \equiv \dfrac{a^2+1}{2a}$, where $a > 0$, **[3]**

(b) $\cosh x \cosh y - \sinh x \sinh y \equiv \cosh(x-y)$. **[3]**

(ii) Use part **(i)(b)** to show that $\cosh^2 x - \sinh^2 x \equiv 1$. **[1]**

(iii) Given that $R > 0$ and $a > 1$, find R and a such that

$$13\cosh x - 5\sinh x \equiv R\cosh(x - \ln a).$$ **[5]**

(iv) Hence write down the coordinates of the minimum point on the curve with equation $y = 13\cosh x - 5\sinh x$. **[2]**

9 **(i)** It is given that, for non-negative integers n,

$$I_n = \int_0^{\frac{1}{2}\pi} \sin^n\theta\,d\theta.$$

Show that, for $n \geqslant 2$,

$$nI_n = (n-1)I_{n-2}.$$ **[4]**

(ii) The equation of a curve, in polar coordinates, is

$$r = \sin^3\theta, \quad \text{for } 0 \leqslant \theta \leqslant \pi.$$

(a) Find the equations of the tangents at the pole and sketch the curve. **[4]**

(b) Find the exact area of the region enclosed by the curve. **[6]**

There are no questions printed on this page.