GCE

Mathematics

Advanced GCE

Unit **4724:** Core Mathematics 4

Mark Scheme for June 2013

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. Annotations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
AG	Answer Given in question
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. **Subject-specific Marking Instructions for GCE Mathematics Pure strand**

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$\frac{(x-7)(x-2)}{(x+2)(x-1)^2} \equiv \frac{A}{x+2} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$ [If no partial fractions seen anywhere, B0]	B1	**SC** $\frac{(x-7)(x-2)}{(x+2)(x-1)^2} \equiv \frac{A}{x+2} + \frac{Bx+C}{(x-1)^2}$ [If no partial fractions seen anywhere, B0]	B1
			$(x-7)(x-2) \equiv A(x-1)^2 + B(x+2)(x-1) + C(x+2)$ [Allow careless minor error but not algebraic method error] or any equiv identity such as $\frac{(x-7)(x-2)}{(x-1)^2} \equiv A + \frac{B(x+2)}{(x-1)} + \frac{C(x+2)}{(x-1)^2}$ (or even the identity on the 1st line), in which values of x are substituted (or cfs compared)	M1	$(x-7)(x-2) \equiv A(x-1)^2 + (Bx+C)(x+2)$ [Allow careless minor error but not algebraic method error] or any equivalent identity (as in previous column) (or even the identity on the 1st line), in which values of x are substituted (or cfs compared)	M1
			$A=4, B=-3, C=2$ or $\frac{4}{x+2} - \frac{3}{x-1} + \frac{2}{(x-1)^2}$ ISW The 3 @ A1 are dep on the used identity being correct.	A1,1,1	$A=4,\ B=-3, C=5$ or $\frac{4}{x+2} + \frac{-3x+5}{(x-1)^2}$	A1 This gives max 3/5 for easier case
			<u>Cover-up:</u> A=4,C=2 score B1,B1; $B=-3$ needs M1, then A1	**[5]**		

Question	Answer	Marks	Guidance	
2	$u = \ln 3x$ and $\mathrm{d}v$ or $\frac{\mathrm{d}v}{\mathrm{d}x} = x^8$	M1	integ by parts as far as $\mathrm{f}(x) +/- \int \mathrm{g}(x)(\mathrm{d}x)$	If difficult to assess, x^8 must be integrated, so look for term in x^9
	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln 3x) = \frac{1}{x}$ or $\frac{3}{3x}$	B1	stated or clearly used	
	$\frac{x^9}{9}\ln 3x - \int \frac{x^9}{9}$ their $\frac{\mathrm{d}u}{\mathrm{d}x}(\mathrm{d}x)$ FT	√A1	i.e. correct understanding of 'by parts'…	..even if ln(3x) incorrectly differentiated
	Indication that $\int kx^8 \mathrm{d}x$ is required	M1	i.e. before integrating, product of terms must be taken	The product may already have been indicated on the previous line
	$\frac{x^9}{9}\ln 3x - \frac{x^9}{81}$ or $\frac{1}{9}x^9\left(\ln 3x - \frac{1}{9}\right)$ ISW (+c) <u>cao</u>	A1	$\frac{1}{9}\frac{x^9}{9}$ to be simplif to $\frac{x^9}{81}$; $\frac{3x^9}{243}$ satis	
		[5]		
	<u>If candidate manipulates ln(3x) first of all</u> ln(3x) = ln 3 + ln x	B1		If, however, ln(3x) is said to be ln 3.ln x, then B0 followed by possible M1 A1 A1 in line with alternative solution on LHS, where the 'M' mark is for dealing with $\int x^8 \ln x \, \mathrm{d}x$ 'by parts' in the right order and the 2 @ A1 are for correct results.
	$u = \ln x$ and $\mathrm{d}v = x^8$	M1	In order to find $\int x^8 \ln x \, \mathrm{d}x$:	
	$\frac{x^9}{9}\ln x - \int \frac{x^9}{9}.\frac{1}{x}(\mathrm{d}x)$ or better	A1		
	$\frac{x^9}{9}\ln x - \frac{x^9}{81}$	A1		
	Their $\int x^8 \ln x \, \mathrm{d}x + \frac{x^9}{9}\ln 3$ (+ c) FT ISW	√A1		

Question			Answer	Marks	Guidance	
3			Set up the 3 relevant equations $1+2\lambda=\mu-1$ $\quad -\lambda=5-\mu$ $\quad 3+5\lambda=2-5\mu$	M1	'M' mark so intention must be clear; minor error(s) only accepted	MR must be consistent; correct version anywhere $\Rightarrow$ not MR
			Attempt to find λ or μ from 2 of the equations & then find μ or λ from any of the 3 equations.	M1	Or find λ, say, from (i)(ii) & then from (ii)(iii) [values shown at next stage] – inconsistency dep*A1 also awarded here	
			$(\lambda,\mu)=$ $(3,8)$ or $(-2\frac{3}{5},2\frac{2}{5})$ or $(-\frac{11}{15},\frac{8}{15})$ or $(3,-3\frac{1}{5})$ or $(-\frac{11}{15},4\frac{4}{15})$ or $(-2\frac{3}{5},-3\frac{1}{5})$ or $(\frac{1}{5},2\frac{2}{5})$ or $(-8\frac{1}{5},8)$ or $(-4\frac{7}{15},\frac{8}{15})$	A1	Accept equivalent proper/improper fractional values or decimal equivalent values	These are all of the solutions obtainable using different combinations of the 3 equations; e.g. using just i & ii or using i & ii to find λ & iii to find μ
			Demonstrate <u>inconsistency</u> i.e. substitute the <u>correct</u> values into a <u>correct</u> equation (but not the immediate last one used)	M1	e.g. after (3,8), subst in iii & write $3+5\times3\ \neq 2-5\times8$ or $3+5\times3=2-5\times8\ \therefore$ do not intersect	
			State "skew"	A1	Dep on 3 @ M1 + A1	
			(a) Identify direction vectors; (b) state "not identical/same/equal/equiv/multiples" or eval cos(angle) & state $\neq 1(\text{or } -1)$; (c) state "not parallel"	B1	dvs <u>must be identified</u>: $\begin{pmatrix}2\\-1\\5\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$ Accept any vector notation.	
				[6]		

Question			Answer	Marks	Guidance	
4			Use of $\sin 2x = +/-2\sin x\cos x$ or $+/-\cos\left(\frac{\pi}{2}-2x\right)$ *or* $\cos 2x = +/-2\cos^2 x +/-1$ etc	M1	Seen anywhere in the solution	
			$\left(\frac{dy}{dx}=\right)$ $-2\sin 2x(\text{or } -4\sin x\cos x)$; $+2\cos x$	B1,B1		
			their $\frac{dy}{dx}=0$	*M1		
			$\left(\frac{\pi}{2},1\right)$; $\left(\frac{\pi}{6},\frac{3}{2}\right)$ and $\left(\frac{5\pi}{6},\frac{3}{2}\right)$	dep* A1; A1	-1(once) for using degrees in an answer instead of radians. If B0 &/or B0 <u>because of sign error</u>, allow A1 to be awarded for $\left(\frac{\pi}{2},1\right)$	<u>SC</u> If A0 but all 3 x-values are correct, award SC A1 <u>SC</u> B2 for 3 ✓answers without working
				[6]		
5	(i)		$\frac{(1+\tan x)-(1-\tan x)}{(1-\tan x)(1+\tan x)}$	M1	Combine (or write as 2 separate fractions) using common denominator	Accept with/without brackets in num Accept $1-\tan x.1+\tan x$ in denom
			$= \frac{2\tan x}{1-\tan^2 x} = \tan 2x$ Answer Given	A1	$\frac{2\tan x}{1-\tan^2 x}$ essential stage N.B. If $\tan x$ changed into $\frac{\sin x}{\cos x}$ before manipulation, apply same principles	A0 for omission of any necessary brackets
				[2]		

Question			Answer	Marks	Guidance	
5	(ii)		$\int \tan 2x \, dx = \lambda \ln(\sec 2x)$ or $\mu \ln(\cos 2x)$ [= F(x)]	M1		
			$\lambda = \frac{1}{2}$ or $\mu = -\frac{1}{2}$	A1		
			their F[$\frac{\pi}{6}$] – their F[$\frac{\pi}{12}$]	M1	dependent on attempt at integration……..	……i.e. not for $\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{6}\right)$
			$\frac{1}{2}\ln 2 - \frac{1}{2}\ln\frac{2}{\sqrt{3}}$ oe	A1	i.e. any correct but probably unsimplified numerical version	
			$\frac{1}{2}\ln\sqrt{3}$ or $\frac{1}{4}\ln 3$ or $\ln 3^{1/4}$ or $\frac{1}{2}\ln\frac{6}{2\sqrt{3}}$ oe ISW	+A1	i.e. any correct version in the form $a \ln b$	
				[5]		

Question			Answer	Marks	Guidance	
6			Find du in terms of dx (or vv) or $\frac{\mathrm{d}u}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}u}$	M1	An attempt - not necessarily accurate	
			Substitute, changing given integral to $\int \frac{u-1}{u^2}(\mathrm{d}u)$	A1	No evidence of x at this A1 stage	
			Provided of form $\frac{au+b}{u^2}$, <u>either</u> split as $\frac{au}{u^2}+\frac{b}{u^2}\ldots$	M1	<u>or</u> use 'parts' with 'u' = $au+b$, 'dv' = $\frac{1}{u^2}$	
			Integrate as $\ln u+\frac{1}{u}$ or FT as $a\ln u-\frac{b}{u}$ [=F(u)]	√A1	or $-(au+b)\frac{1}{u}+a\ln u$ FT [=G(u)]	
			Re-substitute $u = 1 + \ln x$ in F(u)	M1	Re-substitute $u = 1 + \ln x$ in G(u)	
			$\ln(1+\ln x)+\frac{1}{1+\ln x}$ (+ c) ISW	A1	or $\ln(1+\ln x)-\frac{\ln x}{1+\ln x}$ (+ c) ISW	
				[6]		
			<u>In each part, mark the answers, ignoring the labels</u>		**<u>To invoke MR, evidence must be clear</u>**	
7	(i)		$AB=\sqrt{91}$; $AC=\sqrt{27}$ or $3\sqrt{3}$ ISW	B1; B1	9.54 or 9.539392..; 5.2(0) or 5.1961524..	
			Attempting to use $\overrightarrow{AB}.\overrightarrow{AC}=AB.AC\cos\theta$	M1	or $BC^2=AB^2+AC^2-2AB.AC\cos\theta$	
			angle BAC = 171 (3 sf) or 2.99 (rad) (3 sf) ISW	A1	Final acute answer [8.68 or 0.152] /choice → A0	171 to 171.317 or 2.99
				[4]		
7	(ii)		6**i** + 4**j** – 2**k** or –6**i** – 4**j** +2**k**	B1	seen, irrespective of any labelling	
			$6\times(-1)+4\times(-3)-2\times(-9)=0$ (∴ perpendicular) **AG**	B1	oe using $(6,4,-2)$ or $(-6,-4,2)$ and…	....$(-1,-3,-9)$ or $(1,3,9)$
			$6\times1+4\times1-2\times5=0$ (∴ perpendicular) **AG**	B1	oe using $(6,4,-2)$ or $(-6,-4,2)$ and…	… $(1,1,5)$ or $(-1,-1,-5)$
				[3]		
7	(iii)		(AD =) $\sqrt{56}$ or $2\sqrt{14}$ or 7.48… soi	B1		
			area ABC= ½(their)AB × (their)AC × sin(their)BAC	M1	(✓= 3.74… but M mark, not A)	
			$9.3\le V<9.35$, $9\frac{1}{3}$ ISW	A1	Accept even if (i) angle given as 8.68…..	i.e. the acute version not accepted in (i)
				[3]		

Question			Answer	Marks	Guidance	
8	(i)		$\frac{dr}{dt} = \frac{k}{\sqrt{r}}$ oe	B2	B1 for $\frac{dr}{dt} =$; B1 for $\frac{k}{\sqrt{r}}$	SR: B1 for $\frac{dr}{dt} \propto \frac{1}{\sqrt{r}}$
			Sep variables of their diff eqn (or invert) & integrate each side, increasing powers by 1 (or $\frac{1}{r} \to \ln r$)	*M1	their d.e. must be $\frac{dr}{dt}$ (or $\frac{dt}{dr}$) = f(r)	Ignore absence of '+c' after integration
			Subst $\frac{dr}{dt} = 1.08, r = 9$ into their diff eqn to find k	M1	their d.e. must include $\frac{dr}{dt}$ (or $\frac{dt}{dr}$), r & k	(✓$k = 3.24$ but M mark, not A)
			Substitute $t = 5$, $r = 9$ to find 'c'	dep*M1	Must involve '+c' here	
			Correct value of c (probably = 1.8 or -1.8)	A1	Check other values	
			$r = (4.86t + 2.7)^{\frac{2}{3}}$ ISW	A1	Answer required in form $r = f(t)$	
				[7]		
8	(ii)		subst $t = 0$ into any version of (i) result to find finite r	M1		(✓$r \approx 1.938991\ldots$but M mark, not A)
			Any V in range $30.5 \le V < 30.55$, but not fortuitously	A1	Accept 9.72π or $\frac{243}{25}\pi$	
				[2]		

Question			Answer	Marks	Guidance	
9	(i)		$\frac{dy}{dt} = 2(+) - \frac{2}{t^3}$; $\frac{dx}{dt} = -\frac{1}{t^2}$ oe soi ISW	B1, B1		
			$\frac{2}{t} - 2t^2$ or $-2\left(t^2 - \frac{1}{t}\right), \frac{2t^3-2}{-t}, -t^2\left(2 - \frac{2}{t^3}\right)$ oe	B1	ISW. Must not involve (implied)'triple-deckers' e.g. fractions with neg powers…	… e.g. $\frac{2-2t^{-3}}{-t^2}$
				[3]		
9	(ii)		(Any of their expressions for $\frac{dy}{dx}$) = 0 or their $\frac{dy}{dt} = 0$	M1		
			$t = 1 \to$ (stationary point) = (0, 3)	A1	Not awarded if $\frac{dy}{dx}$ is wrong in (i) and used here BUT allow recovery if only explicitly considering $\frac{dy}{dt} = 0$	
			Consider values of x on each side of their critical value of x which lead to finite values of $\frac{dy}{dx}$	M1		
			Hence (0, 3) is a minimum point www	A1	Totally satis; values of x must be close to 0 & not going below or equal to $x = -1$	
				[4]		
9	(iii)		Attempt to find t from $x = \frac{1}{t} - 1$ and substitute into the equation for y	M1		
			$y = \frac{2}{x+1} + (x+1)^2$ oe (can be unsimplified) ISW	A1		
				[2]		

Question			Answer	Marks	Guidance	
10	(i)		$(1-x)^{-3} = 1 + -3.-x + \frac{-3.-4}{2}(-x)^2 + \ldots$ oe; accept $3x$ for $-3.-x$ &/or $-x^2$ or $(x)^2$ for $(-x)^2$ multiplication by x to produce **AG** (**A**nswer **G**iven)	M1 A1 **[2]**	As result is given, this expansion must be shown and then simplified. It must not just be stated as $1+3x+6x^2+\ldots$	For alternative methods such as expanding $(1-x)^3$ and multiplying by $x+3x^2+6x^3$ <u>or</u> using long division, consult TL
10	(ii)		Clear indication that $x = 0.1$ is to be substituted (estimated value is) $0.1+3(0.1)^2+6(0.1)^3 = \underline{0.136}$	M1 A1 **[2]**	e.g. $0.1+3(0.1)^2+6(0.1)^3$ stated	Calculator value $\rightarrow$ M0 (0.13717… is calculator value of $\frac{100}{729}$)
10	(iii)		Sight of $1-x = x\left(\frac{1}{x}-1\right)$ or $1-x = -x\left(1-\frac{1}{x}\right)$ or $\left(\frac{1}{x}-1\right)^3 = -\left(1-\frac{1}{x}\right)^3$ or $(\frac{1}{x}-1)^{-3} = -\left(1-\frac{1}{x}\right)^{-3}$ or $\left(\frac{1}{x}-1\right)^{-3} = -\left(1-\frac{1}{x}\right)^{-3}$ or equivalent Complete satisfactory explanation (no reference to style) www $[1+(-3)(-\frac{1}{x})+\frac{(-3)(-4)}{2}\left(-\frac{1}{x}\right)^2+\ldots]$ $\rightarrow -\frac{1}{x^2}-\frac{3}{x^3}-\frac{6}{x^4}$	B1 B1 M1 A1 **[4]**	(Answer Given) Simplified expansion may be quoted – it may have come from result in part (i). Answer for this expansion is not **AG**.	

Question			Answer	Marks	Guidance	
10	**(iv)**		Must say "Not suitable" and one of following: Either: requires $\left\|\frac{1}{x}\right\| < 1$, which is not true if $x = 0.1$ Or: substitution of positive/small value of x in the expansion gives a negative/large value (which cannot be an approximation to 100/729).	B1 **[1]**	<u>This B1 is dep on $x = 0.1$ used in (ii).</u> Or "because $\frac{1}{x} > 1$" Or "it gives -63100"	Realistic reason If choice given, do not ignore incorrect comments, but ignore irrelevant/unhelpful ones

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