4725 Further Pure Mathematics 1

1 (i) $\begin{pmatrix} 1 & 1 \\ 5 & -1 \end{pmatrix}$

B1 Two elements correct

B1 All four elements correct

2

(ii) *EITHER*

$\frac{1}{3}\begin{pmatrix} 2 & -1 \\ -5 & 4 \end{pmatrix}$

B1 Both diagonals correct

B1 Divide by determinant

2

OR

B1 Solve sim. eqns. 1st column correct

B1 2nd column correct

2 (i) 5

0.927 or 53.1^{o}

B1 Correct modulus

B1 Correct argument, any equivalent form

2

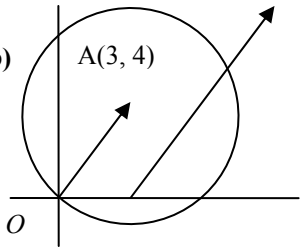
(ii)(a)

B1 Circle centre *A* (3, 4)

B1 Through *O*, allow if centre is (4, 3)

2

(b)



B1 Half line with +ve slope

B1 Starting at (3, 0)

B1 Parallel to *OA*, (implied by correct arg shown)

3

3 (i) $\frac{r}{(r+1)!}$

M1 Common denominator of $(r + 1)!$ or $r!(r + 1)!$

A1 Obtain given answer correctly

2

(ii) $1-\frac{1}{(n+1)!}$

M1 Express terms as differences using (i)

A1 At least 1st two and last term correct

M1 Show pairs cancelling

A1 Correct answer a.e.f.

4

4

B1 Establish result is true, for $n = 1$ (or 2 or 3)

M1 Attempt to multiply **A** and $\mathbf{A}^{n}$, or vice versa

M1 Correct process for matrix multiplication

A1 Obtain 3^{n+1}, 0 and 1

A1 Obtain $\frac{1}{2}(3^{n+1} - 1)$

A1 Statement of Induction conclusion, only if 5 marks earned, but may be in body of working

6

5			**M1**	Express as difference of two series
			M1	Use standard results
		$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$	**A1**	Correct unsimplified answer
			M1	Attempt to factorise
			A1	At least factor of $n(n+1)$
		$\frac{1}{12}n(n+1)(3n+2)(n-1)$	**A1**	Obtain correct answer
			6	
6	**(i)**	$3 - i$	**B1**	Conjugate stated
			1	
	(ii)	*EITHER*	**M1**	Use sum of roots
			A1	Obtain correct answer
			M1	Use sum of pairs of roots
			A1	Obtain correct answer
			M1	Use product of roots
		$a = -8,\ b = 22,\ c = -20$	**A1**	Obtain correct answers
			6	
		OR	**M1**	Attempt to find a quadratic factor
			A1	Obtain correct factor
			M1	Expand linear and quadratic factors
		$a = -8, b = 22, c = -20$	**A1A1A1**	Obtain correct answers
		OR		
			M1	Substitute 1 imaginary & the real root into eqn
			M1	Equate real and imaginary parts
			M1	Attempt to solve 3 eqns.
		$a = -8, b = 22, c = -20$	**A1A1A1**	Obtain correct answers
7	**(i)**		**B1**	Enlargement (centre O) scale factor 6
			1	
	(ii)		**B1**	Reflection
			B1	Mirror line is $y = x$
			2	
	(iii)		**B1**	Stretch in y direction
			B1	Scale factor 6, must be a stretch
			2	
	(iv)		**B1**	Rotation
			B1	36.9^o clockwise or equivalent
			2	

8	$\alpha+\beta=-k$	**B1**	State or use correct value
	$\alpha\beta=2k$	**B1**	State or use correct value
		M1	Attempt to express sum of new roots in terms of $\alpha+\beta$, $\alpha\beta$
	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	**A1**	Obtain correct expression
	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{1}{2}(k-4)$	**A1**	Obtain correct answer a.e.f.
	$\alpha'\beta'=1$	**B1**	Correct product of new roots seen
	$x^2-\frac{1}{2}(k-4)x+1=0$	**B1ft**	Obtain correct answer, must be an eqn.
		7	
			Alternative for last 5 marks
		M1	Obtain expression for $u=\frac{\alpha}{\beta}$ in terms of k and α or k and β
		A1	Obtain a correct expression
		A1	rearrange to get α in terms of u
		M1	Substitute into given equation
		A1	Obtain correct answer
9	**(i)**	**M1**	Attempt to equate real and imaginary parts of $(x + iy)^2$ and $5 + 12i$
	$x^2 - y^2 = 5$ and $xy = 6$	**A1**	Obtain both results
		M1	Eliminate to obtain a quadratic in x^2 or y^2
	$\pm(3 + 2i)$	**M1**	Solve a 3 term quadratic & obtain x or y
		A1	Obtain correct answers as complex nos.
		5	
	(ii) $5 - 12i$	**B1B1**	Correct real and imaginary parts
		2	
	(iii)	**M1**	Attempt to solve a quadratic equation
	$x^2 = 5 \pm 12i$	**A1**	Obtain correct answers
	$x = \pm(3 \pm 2i)$	**A1A1**	Each pair of correct answers a.e.f.
		4	

10 (i)

M1 Find value of det **AB**
A1 Correct value 2 seen
2

(ii)

$$(\mathbf{AB})^{-1} = \frac{1}{2}\begin{pmatrix} 0 & 3 & -1 \\ 0 & -1 & 1 \\ 2 & 6-3a & a-6 \end{pmatrix}$$

M1 Show correct process for adjoint entries
A1 Obtain at least 4 correct entries in adjoint
B1 Divide by their determinant
A1 Obtain completely correct answer
4

(iii) *EITHER*

$$\mathbf{B}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ -6 & 2 & -2 \end{pmatrix}$$

M1 State or imply $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
A1 Obtain $\mathbf{B}^{-1} = (\mathbf{AB})^{-1} \times \mathbf{A}$
M1 Correct multiplication process seen
A1 Obtain three correct elements
A1 All elements correct
5

OR

M1 Attempt to find elements of **B**
A1 All correct
M1 Correct process for $\mathbf{B}^{-1}$
A1 3 elements correct
A1 All elements correct