OCR
RECOGNISING ACHIEVEMENT

Thursday 13 June 2013 – Morning

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 By using the substitution $t = \tan\frac{1}{2}\theta$, find $\int_0^{\frac{1}{2}\pi} \frac{1}{1+\cos\theta}\,\mathrm{d}\theta$. **[5]**

2 **(i)** Using the definitions for $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x}, show that $\cosh^2 x - \sinh^2 x \equiv 1$. **[3]**

(ii) Hence solve the equation $\sinh^2 x = 5\cosh x - 7$, giving your answers in logarithmic form. **[5]**

3 It is given that $\mathrm{f}(x) = \tanh^{-1}\left(\frac{1-x}{3+x}\right)$ for $x > -1$.

(i) Show that $\mathrm{f}''(x) = \frac{1}{2(x+1)^2}$. **[6]**

(ii) Hence find the Maclaurin series for $\mathrm{f}(x)$ up to and including the term in x^2. **[4]**

4 It is given that $I_n = \int_0^{\frac{1}{2}\pi} \cos^n x\,\mathrm{d}x$ for $n \geqslant 0$.

(i) Show that $I_n = \frac{n-1}{n} I_{n-2}$ for $n \geqslant 2$. **[5]**

(ii) Hence find I_{11} as an exact fraction. **[3]**

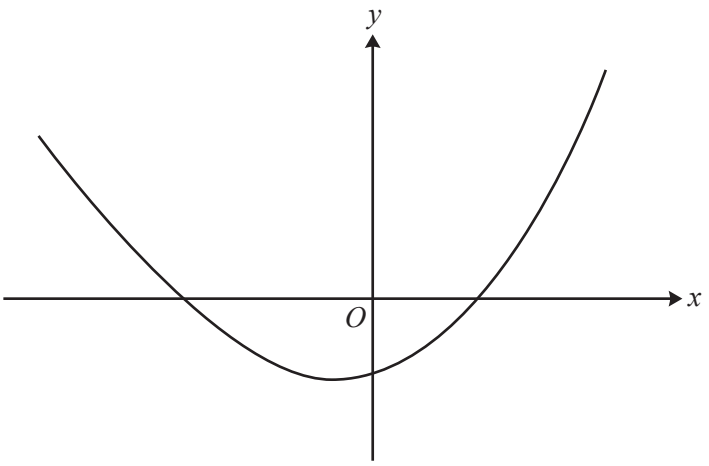
5 You are given that the equation $x^3 + 4x^2 + x - 1 = 0$ has a root, α, where $-1 < \alpha < 0$.

(i) Show that the Newton-Raphson iterative formula for this equation can be written in the form

$$x_{n+1} = \frac{2x_n^3 + 4x_n^2 + 1}{3x_n^2 + 8x_n + 1}.$$ **[3]**

(ii) Using the initial value $x_1 = -0.7$, find x_2 and x_3 and find α correct to 5 decimal places. **[3]**

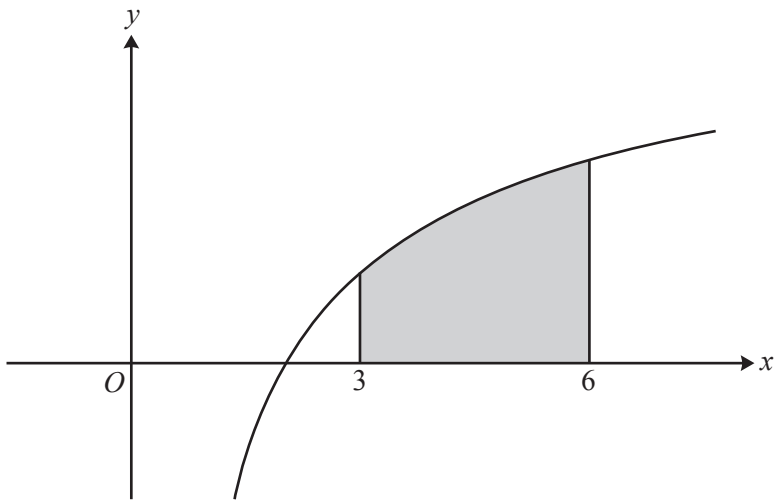
(iii) The diagram shows a sketch of the curve $y = x^3 + 4x^2 + x - 1$ for $-1.5 \leqslant x \leqslant 1$.



Using the copy of the diagram in your answer book, explain why the initial value $x_1 = 0$ will fail to find α. **[2]**

[Questions 6, 7 and 8 are printed overleaf.]

6



The diagram shows part of the curve $y = \ln\big(\ln(x)\big)$. The region between the curve and the x-axis for $3 \leqslant x \leqslant 6$ is shaded.

(i) By considering n rectangles of equal width, show that a lower bound, L, for the area of the shaded region is $\dfrac{3}{n}\displaystyle\sum_{r=0}^{n-1} \ln\left(\ln\left(3 + \frac{3r}{n}\right)\right)$. **[3]**

(ii) By considering another set of n rectangles of equal width, find a similar expression for an upper bound, U, for the area of the shaded region. **[1]**

(iii) Find the least value of n for which $U - L < 0.001$. **[4]**

7 The equation of a curve is $y = \dfrac{x^2 + 1}{(x + 1)(x - 7)}$.

(i) Write down the equations of the asymptotes. **[3]**

(ii) Find the coordinates of the stationary points on the curve. **[5]**

(iii) Find the coordinates of the point where the curve meets one of its asymptotes. **[3]**

(iv) Sketch the curve. **[3]**

8 The equation of a curve is $x^2 + y^2 - x = \sqrt{x^2 + y^2}$.

(i) Find the polar equation of this curve in the form $r = \mathrm{f}(\theta)$. **[3]**

(ii) Sketch the curve. **[2]**

(iii) The line $x + 2y = 2$ divides the region enclosed by the curve into two parts. Find the ratio of the two areas. **[6]**

