OCR RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

4725

Further Pure Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Friday 11 June 2010
Morning

Duration: 1 hour 30 minutes

* 4 7 2 5 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

2R–9K03

1 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$. **[5]**

2 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 1 & -4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ -2 & 2 \end{pmatrix}$. Find

(i) **AB**, **[2]**

(ii) **BA** – 4**C**. **[4]**

3 Find $\sum_{r=1}^{n}(2r-1)^2$, expressing your answer in a fully factorised form. **[6]**

4 The complex numbers a and b are given by $a = 7 + 6\mathrm{i}$ and $b = 1 - 3\mathrm{i}$. Showing clearly how you obtain your answers, find

(i) $|a - 2b|$ and $\arg(a - 2b)$, **[4]**

(ii) $\dfrac{b}{a}$, giving your answer in the form $x + \mathrm{i}y$. **[3]**

5 **(a)** Write down the matrix that represents a reflection in the line $y = x$. **[2]**

(b) Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$, **[2]**

(ii) $\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$. **[2]**

6 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z - 3 + 4\mathrm{i}| = 5$, **[2]**

(b) $|z| = |z - 6|$. **[2]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 + 4\mathrm{i}| \leqslant 5 \quad \text{and} \quad |z| \geqslant |z - 6|.$$ **[2]**

7 The quadratic equation $x^2 + 2kx + k = 0$, where k is a non-zero constant, has roots α and β. Find a quadratic equation with roots $\dfrac{\alpha + \beta}{\alpha}$ and $\dfrac{\alpha + \beta}{\beta}$. **[7]**

8 **(i)** Show that $\dfrac{1}{\sqrt{r+2}+\sqrt{r}} \equiv \dfrac{\sqrt{r+2}-\sqrt{r}}{2}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r+2}+\sqrt{r}}.$$ **[6]**

(iii) State, giving a brief reason, whether the series $\displaystyle\sum_{r=1}^{\infty} \frac{1}{\sqrt{r+2}+\sqrt{r}}$ converges. **[1]**

9 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & a & -1 \\ 0 & a & 2 \\ 1 & 2 & 1 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of $\mathbf{A}$. **[3]**

(ii) Three simultaneous equations are shown below.

$$\begin{aligned} ax + ay - z &= -1 \\ ay + 2z &= 2a \\ x + 2y + z &= 1 \end{aligned}$$

For each of the following values of a, determine whether the equations are consistent or inconsistent. If the equations are consistent, determine whether or not there is a unique solution.

(a) $a = 0$

(b) $a = 1$

(c) $a = 2$

[6]

10 The complex number z, where $0 < \arg z < \frac{1}{2}\pi$, is such that $z^2 = 3 + 4\mathrm{i}$.

(i) Use an algebraic method to find z. **[5]**

(ii) Show that $z^3 = 2 + 11\mathrm{i}$. **[1]**

The complex number w is the root of the equation

$$w^6 - 4w^3 + 125 = 0$$

for which $-\frac{1}{2}\pi < \arg w < 0$.

(iii) Find w. **[5]**