OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4726

Further Pure Mathematics 2

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Monday 20 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Express $\dfrac{2x+3}{(x+3)(x^2+9)}$ in partial fractions. **[5]**

2 A curve has equation $y = \dfrac{x^2-6x-5}{x-2}$.

(i) Find the equations of the asymptotes. **[3]**

(ii) Show that y can take all real values. **[4]**

3 It is given that $\mathrm{F}(x) = 2 + \ln x$. The iteration $x_{n+1} = \mathrm{F}(x_n)$ is to be used to find a root, α, of the equation $x = 2 + \ln x$.

(i) Taking $x_1 = 3.1$, find x_2 and x_3, giving your answers correct to 5 decimal places. **[2]**

(ii) The error e_n is defined by $e_n = \alpha - x_n$. Given that $\alpha = 3.146\,19$, correct to 5 decimal places, use the values of e_2 and e_3 to make an estimate of $\mathrm{F}'(\alpha)$ correct to 3 decimal places. State the true value of $\mathrm{F}'(\alpha)$ correct to 4 decimal places. **[3]**

(iii) Illustrate the iteration by drawing a sketch of $y = x$ and $y = \mathrm{F}(x)$, showing how the values of x_n approach α. State whether the convergence is of the 'staircase' or 'cobweb' type. **[3]**

4 A curve C has the cartesian equation $x^3 + y^3 = axy$, where $x \geqslant 0$, $y \geqslant 0$ and $a > 0$.

(i) Express the polar equation of C in the form $r = \mathrm{f}(\theta)$ and state the limits between which θ lies. **[3]**

The line $\theta = \alpha$ is a line of symmetry of C.

(ii) Find and simplify an expression for $\mathrm{f}\left(\frac{1}{2}\pi - \theta\right)$ and hence explain why $\alpha = \frac{1}{4}\pi$. **[3]**

(iii) Find the value of r when $\theta = \frac{1}{4}\pi$. **[1]**

(iv) Sketch the curve C. **[2]**

5 **(i)** Prove that, if $y = \sin^{-1} x$, then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{1-x^2}}$. **[3]**

(ii) Find the Maclaurin series for $\sin^{-1} x$, up to and including the term in x^3. **[5]**

(iii) Use the result of part **(ii)** and the Maclaurin series for $\ln(1+x)$ to find the Maclaurin series for $(\sin^{-1} x)\ln(1+x)$, up to and including the term in x^4. **[4]**

6 It is given that $I_n = \displaystyle\int_0^1 x^n(1-x)^{\frac{3}{2}}\,\mathrm{d}x$, for $n \geqslant 0$.

(i) Show that $I_n = \dfrac{2n}{2n+5} I_{n-1}$, for $n \geqslant 1$. **[6]**

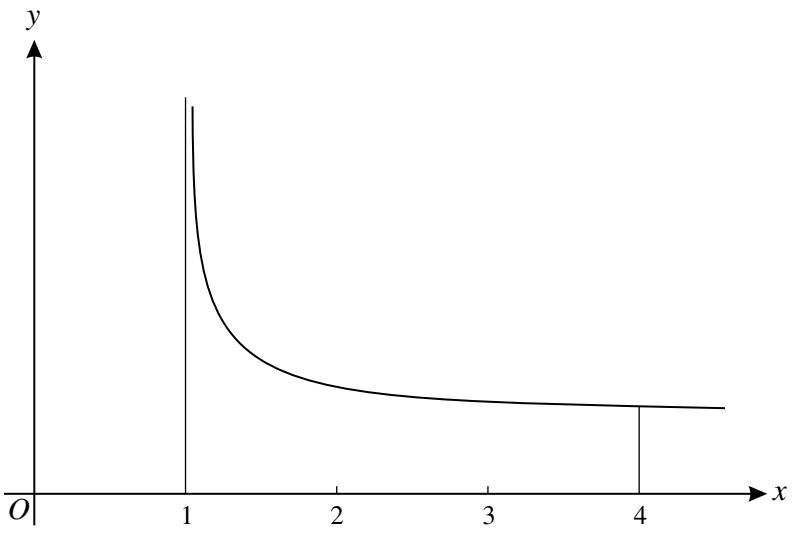
(ii) Hence find the exact value of I_3. **[4]**

7 **(i)** Sketch the graph of $y = \tanh x$ and state the value of the gradient when $x = 0$. On the same axes, sketch the graph of $y = \tanh^{-1} x$. Label each curve and give the equations of the asymptotes. **[4]**

(ii) Find $\int_0^k \tanh x \, \mathrm{d}x$, where $k > 0$. **[2]**

(iii) Deduce, or show otherwise, that $\int_0^{\tanh k} \tanh^{-1} x \, \mathrm{d}x = k \tanh k - \ln(\cosh k)$. **[4]**

8 **(i)** Use the substitution $x = \cosh^2 u$ to find $\int \sqrt{\frac{x}{x-1}} \, \mathrm{d}x$, giving your answer in the form $\mathrm{f}(x) + \ln\big(\mathrm{g}(x)\big)$. **[7]**



(ii) Hence calculate the exact area of the region between the curve $y = \sqrt{\frac{x}{x-1}}$, the x-axis and the lines $x = 1$ and $x = 4$ (see diagram). **[1]**

(iii) What can you say about the volume of the solid of revolution obtained when the region defined in part **(ii)** is rotated completely about the x-axis? Justify your answer. **[3]**

