Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MM03

Unit Mechanics 3

Tuesday 18 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A stone, of mass 2 kg, is moving in a straight line on a smooth horizontal sheet of ice under the action of a single force which acts in the direction of motion. At time t seconds, the force has magnitude $(3t + 1)$ newtons, $0 \leqslant t \leqslant 3$.

When $t = 0$, the stone has velocity $1\,\mathrm{m\,s^{-1}}$.
When $t = T$, the stone has velocity $5\,\mathrm{m\,s^{-1}}$.

Find the value of T. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 1

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QUESTION PART REFERENCE

Answer space for question 1

2 A car has mass m and travels up a slope which is inclined at an angle θ to the horizontal. The car reaches a maximum speed v at a height h above its initial position. A constant resistance force R opposes the motion of the car, which has a maximum engine power output P.

Neda finds a formula for P as

$$P = mgv\sin\theta + Rv + \frac{1}{2}mv^3\frac{\sin\theta}{h}$$

where g is the acceleration due to gravity.

Given that the engine power output may be measured in newton metres per second, determine whether the formula is dimensionally consistent. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 2

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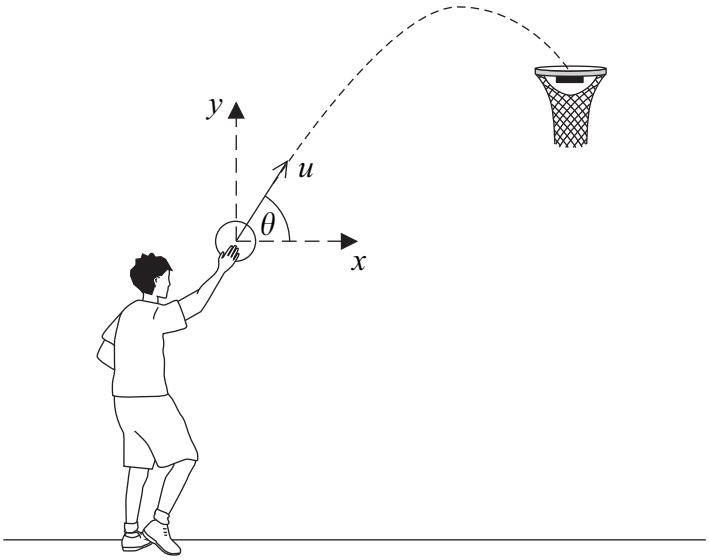
3 A player projects a basketball with speed $u\,\text{m s}^{-1}$ at an angle θ above the horizontal. The basketball travels in a vertical plane through the point of projection and goes into the basket. During the motion, the horizontal and upward vertical displacements of the basketball from the point of projection are x metres and y metres respectively.



(a) Find an expression for y in terms of x, u, g and $\tan\theta$. *(6 marks)*

(b) The player projects the basketball with speed $8\,\text{m s}^{-1}$ from a point 0.5 metres vertically below and 5 metres horizontally from the basket.

(i) Show that the two possible values of θ are approximately 63.1° and 32.6°, correct to three significant figures. *(5 marks)*

(ii) Given that the player projects the basketball at 63.1° to the horizontal, find the direction of the motion of the basketball as it enters the basket. Give your answer to the nearest degree. *(4 marks)*

(c) State a modelling assumption needed for answering parts **(a)** and **(b)** of this question. *(1 mark)*

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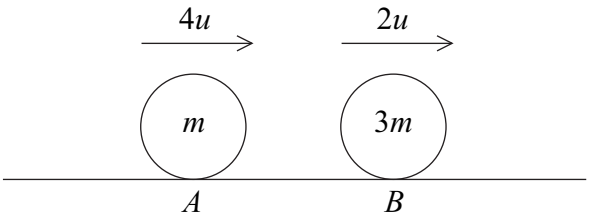
4 A smooth sphere A, of mass m, is moving with speed $4u$ in a straight line on a smooth horizontal table. A smooth sphere B, of mass $3m$, has the same radius as A and is moving on the table with speed $2u$ in the same direction as A.



The sphere A collides directly with sphere B. The coefficient of restitution between A and B is e.

(a) Find, in terms of u and e, the speeds of A and B immediately after the collision. *(6 marks)*

(b) Show that the speed of B after the collision cannot be greater than $3u$. *(2 marks)*

(c) Given that $e = \frac{2}{3}$, find, in terms of m and u, the magnitude of the impulse exerted on B in the collision. *(3 marks)*

QUESTION PART REFERENCE

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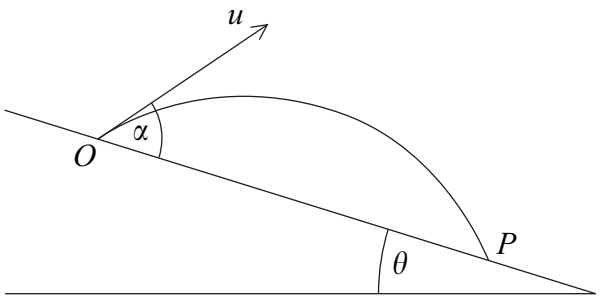
5 A particle is projected from a point O on a plane which is inclined at an angle θ to the horizontal. The particle is projected down the plane with velocity u at an angle α above the plane. The particle first strikes the plane at a point P, as shown in the diagram. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Given that the time of flight from O to P is T, find an expression for u in terms of θ, α, T and g. *(4 marks)*

(b) Using the identity $\cos(X - Y) = \cos X \cos Y + \sin X \sin Y$, show that the distance OP is given by $\dfrac{2u^2 \sin\alpha \cos(\alpha - \theta)}{g \cos^2 \theta}$. *(6 marks)*

QUESTION PART REFERENCE

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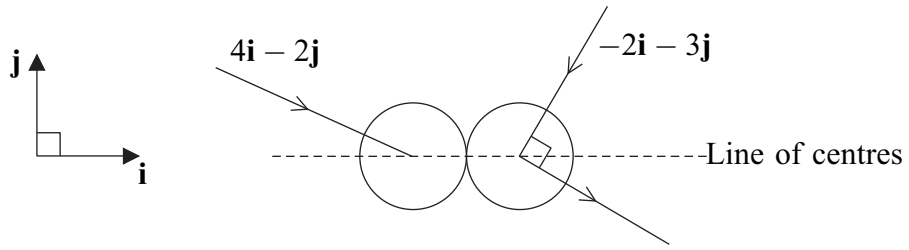
6 Two smooth spheres, A and B, have equal radii and masses 4 kg and 2 kg respectively. The sphere A is moving with velocity $(4\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$ and the sphere B is moving with velocity $(-2\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-1}}$ on the same smooth horizontal surface. The spheres collide when their line of centres is parallel to unit vector $\mathbf{i}$. The direction of motion of B is changed through 90° by the collision, as shown in the diagram.



(a) Show that the velocity of B immediately after the collision is $(\frac{9}{2}\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-1}}$. *(4 marks)*

(b) Find the coefficient of restitution between the spheres. *(5 marks)*

(c) Find the impulse exerted on B during the collision. State the units of your answer. *(3 marks)*

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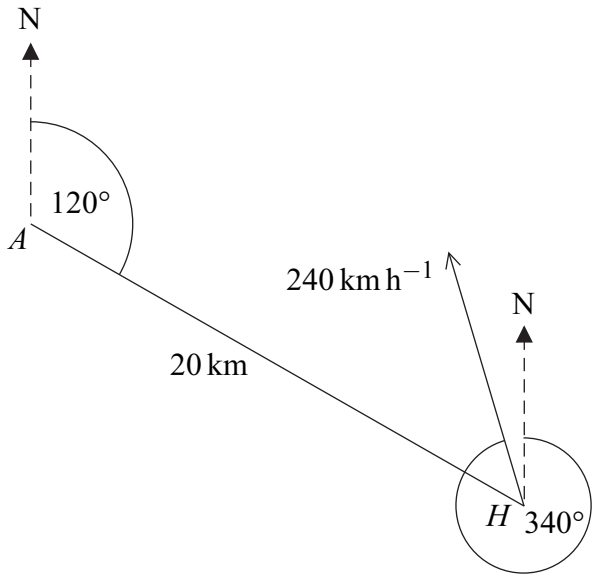
7 From an aircraft A, a helicopter H is observed 20 km away on a bearing of 120°. The helicopter H is travelling horizontally with a constant speed 240 km h^{-1} on a bearing of 340°. The aircraft A is travelling with constant speed v_A km h^{-1} in a straight line and at the same altitude as H.



(a) Given that $v_A = 200$:

(i) find a bearing, to one decimal place, on which A could travel in order to intercept H; *(5 marks)*

(ii) find the time, in minutes, that it would take A to intercept H on this bearing. *(4 marks)*

(b) Given that $v_A = 150$, find the bearing on which A should travel in order to approach H as closely as possible. Give your answer to one decimal place. *(5 marks)*

QUESTION PART REFERENCE

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END OF QUESTIONS