

1.

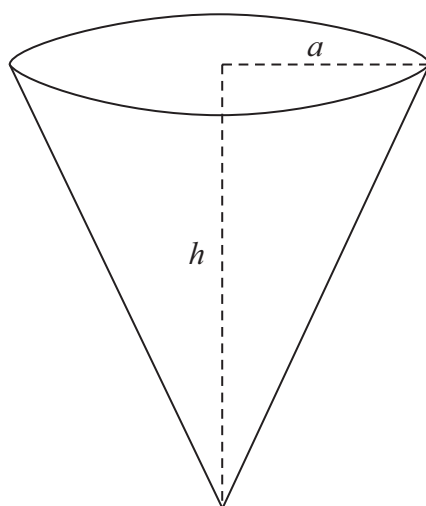


Figure 1

A hollow right circular cone, of base radius a and height h , is fixed with its axis vertical and vertex downwards, as shown in Figure 1. A particle moves with constant speed v in a horizontal circle of radius $\frac{1}{3}a$ on the smooth inner surface of the cone.

Show that $v = \sqrt{\left(\frac{1}{3}hg\right)}$. (7)



5.

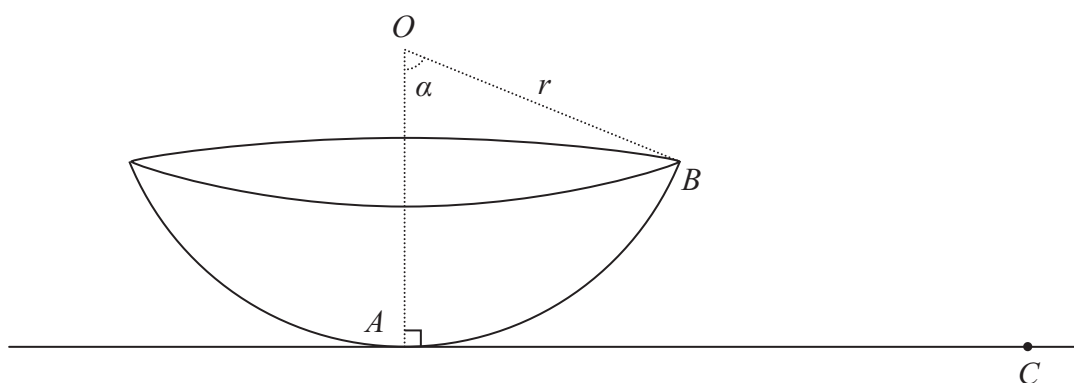


Figure 2

Part of a hollow spherical shell, centre O and radius r , forms a bowl with a plane circular rim. The bowl is fixed to a horizontal surface at A with the rim uppermost and horizontal.

The point A is the lowest point of the bowl. The point B , where $\angle AOB = \alpha$ and $\tan \alpha = \frac{3}{4}$, is on the rim of the bowl, as shown in Figure 2. A small smooth marble M is placed inside the bowl at A , and given an initial horizontal speed \sqrt{gr} . The motion of M takes place in the vertical plane OAB .

- (a) Show that the speed of M as it reaches B is $\sqrt{\left(\frac{3}{5}gr\right)}$. **(4)**

After leaving the surface of the bowl at B , M moves freely under gravity and first strikes the horizontal surface at the point C . Given that $r = 0.4\text{ m}$,

- (b) find the distance AC . **(8)**

6. (a) A uniform lamina is in the shape of a quadrant of a circle of radius a . Show, by integration, that the centre of mass of the lamina is at a distance of $\frac{4a}{3\pi}$ from each of its straight edges. (7)

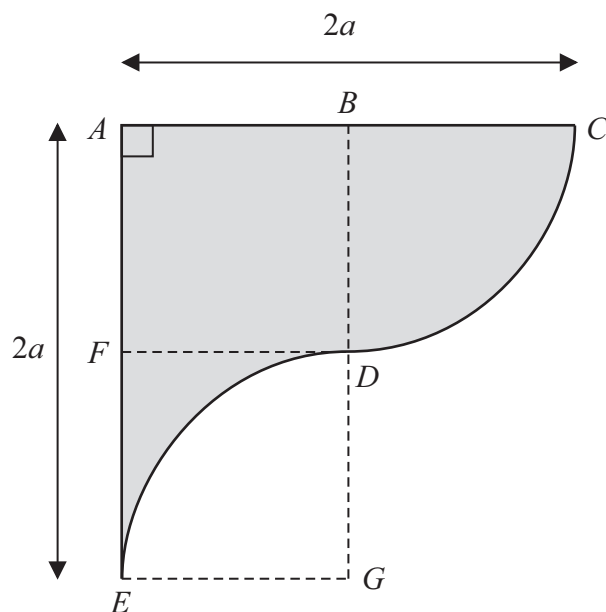


Figure 3

A second uniform lamina $ABCDEFA$ is shown shaded in Figure 3. The straight sides AC and AE are perpendicular and $AC = AE = 2a$. In the figure, the midpoint of AC is B , the midpoint of AE is F , and $ABDF$ and $DGEF$ are squares of side a . BCD is a quadrant of a circle with centre B . DGE is a quadrant of a circle with centre G .

- (b) Find the distance of the centre of mass of the lamina from the side AE . (5)

The lamina is smoothly hinged to a horizontal axis which passes through E and is perpendicular to the plane of the lamina. The lamina has weight W newtons. The lamina is held in equilibrium in a vertical plane, with A vertically above E , by a horizontal force of magnitude X newtons applied at C .

- (c) Find X in terms of W . (3)



7. Two points A and B are 4 m apart on a smooth horizontal surface. A light elastic string, of natural length 0.8 m and modulus of elasticity 15 N, has one end attached to the point A . A light elastic string, of natural length 0.8 m and modulus of elasticity 10 N, has one end attached to the point B . A particle P of mass 0.2 kg is attached to the free end of each string. The particle rests in equilibrium on the surface at the point C on the straight line between A and B .

(a) Show that the length of AC is 1.76 m. (4)

The particle P is now held at the point D on the line AB such that $AD = 2.16$ m. The particle is then released from rest and in the subsequent motion both strings remain taut.

(b) Show that P moves with simple harmonic motion. (4)

(c) Find the speed of P as it passes through the point C . (2)

(d) Find the time from the instant when P is released from D until the instant when P is first moving with speed 2 m s^{-1} . (4)



