Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MD02

Unit Decision 2

Wednesday 24 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 **Figure 2**, on the page opposite, shows an activity diagram for a project. Each activity requires one worker. The duration required for each activity is given in hours.

(a) On **Figure 1** below, complete the precedence table.

[1 mark]

(b) Find the earliest start time and the latest finish time for each activity and insert their values on **Figure 2**.

[4 marks]

(c) List the critical paths.

[2 marks]

(d) Find the float time of activity E.

[1 mark]

(e) Using **Figure 3** opposite, draw a Gantt diagram to illustrate how the project can be completed in the minimum time, assuming that each activity is to start as early as possible.

[3 marks]

(f) Given that there is only one worker available for the project, find the minimum completion time for the project.

[1 mark]

(g) Given that there are two workers available for the project, find the minimum completion time for the project. Show a suitable allocation of tasks to the two workers.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

Figure 1

Activity	Immediate predecessor(s)
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	

QUESTION PART REFERENCE

Answer space for question 1

Figure 2

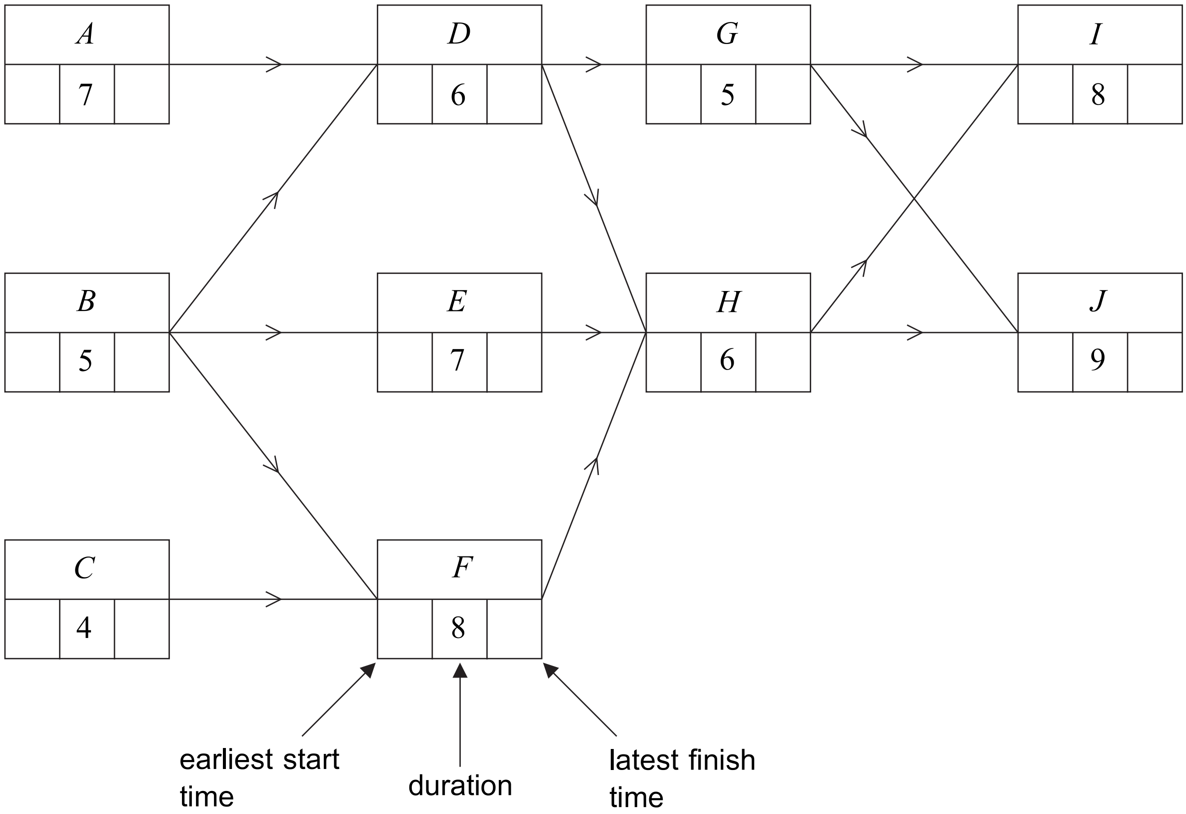
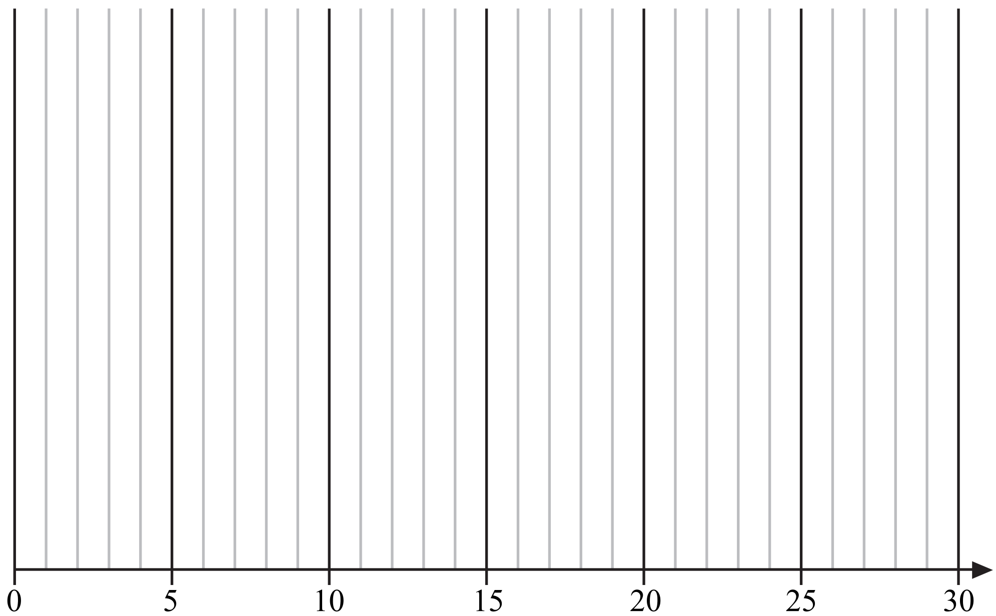


Figure 3



Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 1

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 1

Turn over ▶

2 Stan and Christine play a zero-sum game. The game is represented by the following pay-off matrix for Stan.

Christine

	Strategy	**D**	**E**	**F**	**G**
Stan	**A**	3	−3	−1	0
	B	−1	−4	2	3
	C	1	0	−3	−2

(a) Find the play-safe strategy for each player.

[3 marks]

(b) Show that there is no stable solution.

[1 mark]

(c) Explain why a suitable pay-off matrix **for Christine** is given by

3	4	0
1	−2	3

[4 marks]

QUESTION PART REFERENCE | **Answer space for question 2**

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 2

Turn over ▶

Do not write outside the box

3 In the London 2012 Olympics, the Jamaican 4×100 metres relay team set a world record time of 36.84 seconds.

Athletes take different times to run each of the four legs.

The coach of a national athletics team has five athletes available for a major championship. The lowest times that the five athletes take to cover each of the four legs is given in the table below.

The coach is to allocate a different athlete from the five available athletes, A, B, C, D and E, to each of the four legs to produce the lowest total time.

	Leg 1	**Leg 2**	**Leg 3**	**Leg 4**
Athlete *A*	9.84	8.91	8.98	8.70
Athlete *B*	10.28	9.06	9.24	9.05
Athlete *C*	10.31	9.11	9.22	9.18
Athlete *D*	10.04	9.07	9.19	9.01
Athlete *E*	9.91	8.95	9.09	8.74

Use the Hungarian algorithm, by reducing the **columns first**, to assign an athlete to each leg so that the total time of the four athletes is minimised.

State the allocation of the athletes to the four legs and the total time.

[11 marks]

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

4 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 2x + 3y + 4z$

subject to

$$x + y + 2z \leqslant 20$$
$$3x + 2y + z \leqslant 30$$
$$2x + 3y + z \leqslant 40$$

and $x \geqslant 0\,,\ y \geqslant 0\,,\ z \geqslant 0$

[2 marks]

(b) (i) The first pivot to be chosen is from the z-column. Identify the pivot and explain why this particular value is chosen.

[2 marks]

(ii) Perform one iteration of the Simplex method.

[3 marks]

(c) (i) Perform one further iteration.

[3 marks]

(ii) Interpret your final tableau and state the values of your slack variables.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 4

Do not write outside the box

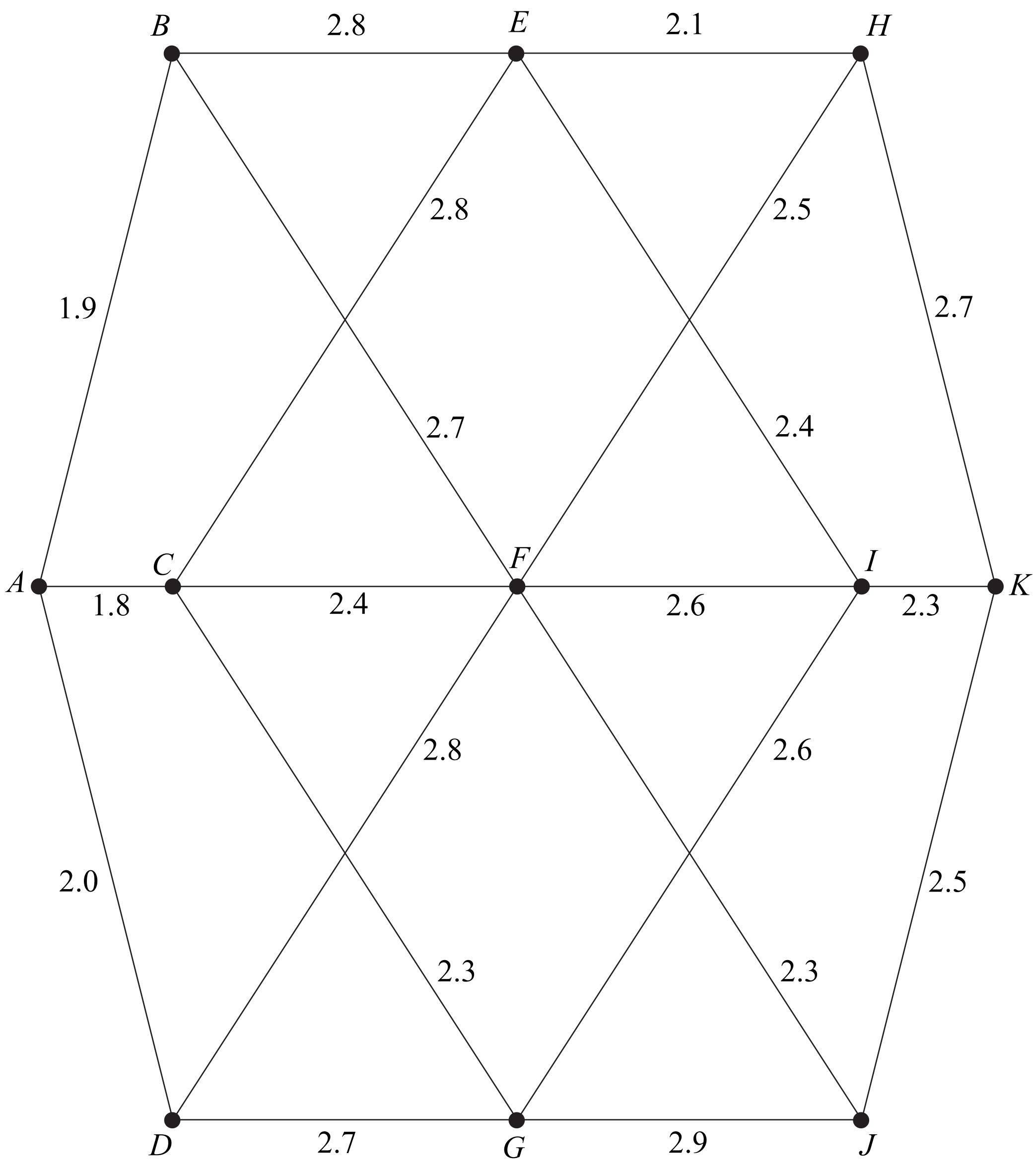
QUESTION PART REFERENCE

Answer space for question 4

5 Tom is going on a driving holiday and wishes to drive from A to K.

The network below shows a system of roads. The number on each edge represents the maximum altitude of the road, in hundreds of metres above sea level.

Tom wants to ensure that the maximum altitude of any road along the route from A to K is minimised.



(a) **Working backwards from K**, use dynamic programming to find the optimal route when driving from A to K.

You must complete the table opposite as your solution.

[9 marks]

(b) Tom finds that the road CF is blocked. Find Tom's new optimal route and the maximum altitude of any road on this route.

[2 marks]

Do not write outside the box

Answer space for question 5

Stage	State	From	Value
1	*H*	*K*	
	I	*K*	
	J	*K*	
2			

(a) Optimal route is ....

(b) Tom's route is ....

Maximum altitude is ....

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 5

Do not write outside the box

QUESTION PART REFERENCE

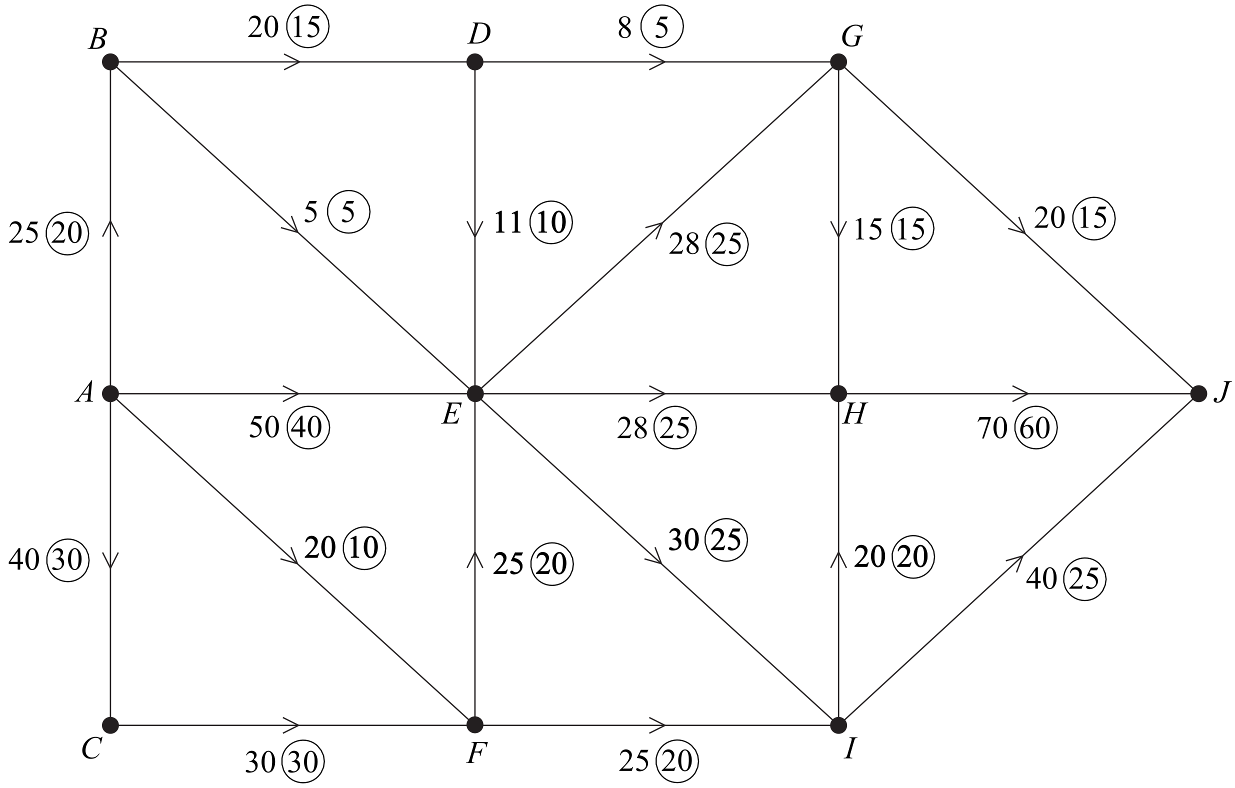
Answer space for question 5

Turn over ▶

6 **Figure 4** below shows a network of pipes.

The capacity of each pipe is given by the number **not circled** on each edge.
The numbers in circles represent an initial flow.

Figure 4



(a) Find the value of the initial flow.

[1 mark]

(b) (i) Use the initial flow and the labelling procedure on **Figure 5** to find the maximum flow through the network. You should indicate any flow-augmenting routes in the table and modify the potential increases and decreases of the flow on the network.

[5 marks]

(ii) State the value of the maximum flow and, on **Figure 6**, illustrate a possible flow along each edge corresponding to this maximum flow.

[2 marks]

(c) Confirm that you have a maximum flow by finding a cut of the same value. List the edges of your cut.

[2 marks]

(d) On a particular day, there is a restriction at vertex G which allows a maximum flow through G of 30.

Find, by inspection, the maximum flow through the network on this day.

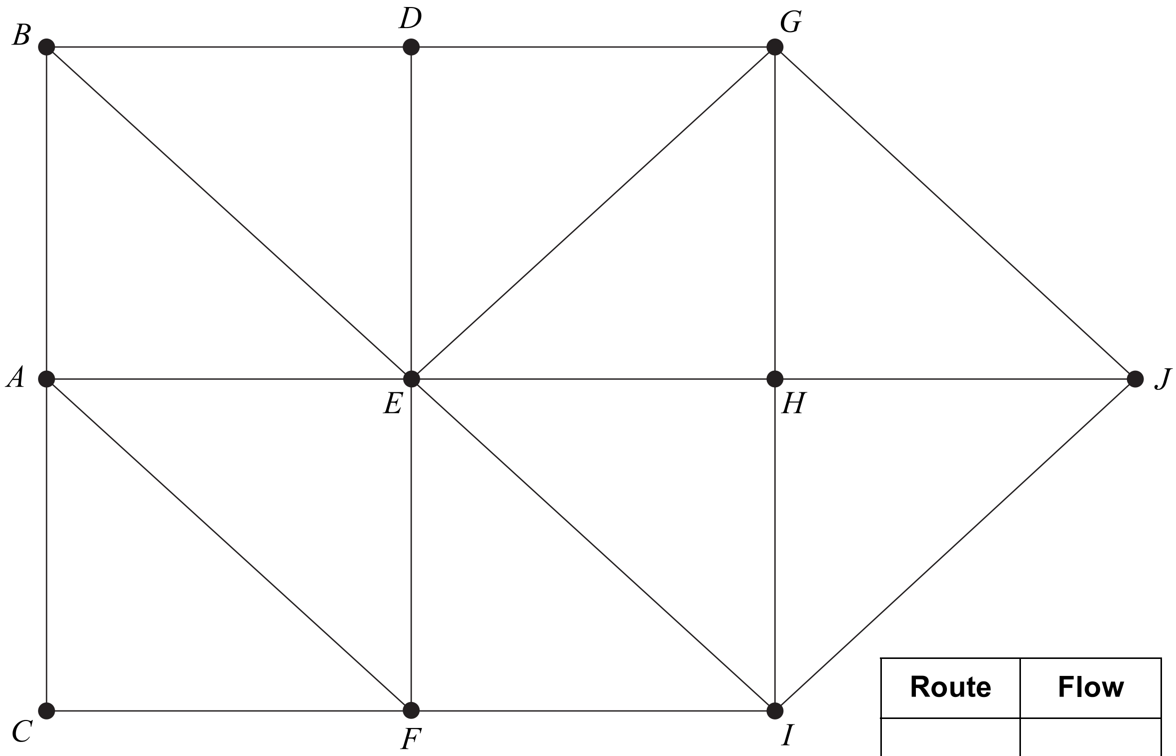
[2 marks]

QUESTION PART REFERENCE

Answer space for question 6

(a) Initial flow = ...

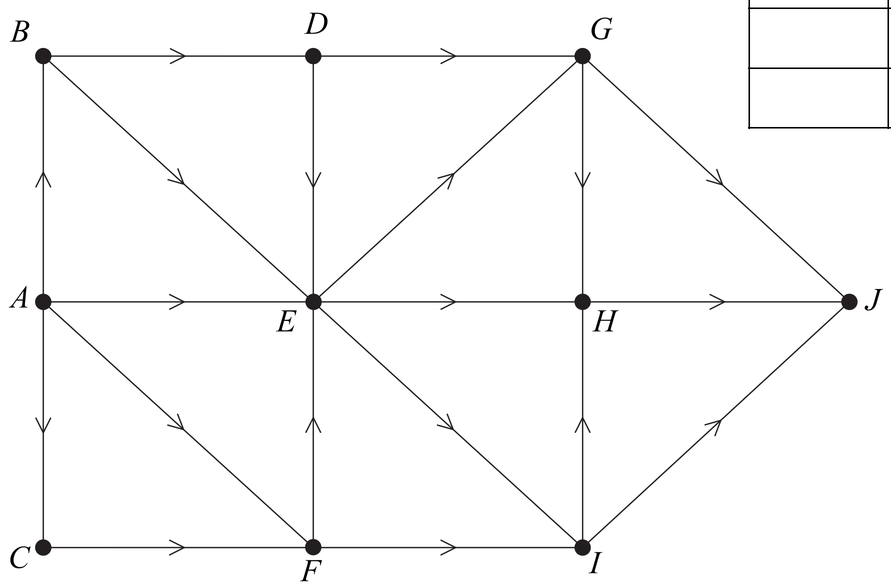
(b)(i) **Figure 5**



Route	Flow

(b)(ii) Maximum flow = ...

Figure 6



Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 6

Turn over ▶

7 Arsene and Jose play a zero-sum game. The game is represented by the following pay-off matrix for Arsene, where x is a constant.

The value of the game is 2.5.

Jose

Arsene

Strategy	C	D
A	$x+3$	1
B	$x+1$	3

(a) Find the optimal mixed strategy for Arsene.

[4 marks]

(b) Find the value of x.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 7

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 7

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 7

END OF QUESTIONS