

2

- 1 Find the determinant of the matrix $\begin{pmatrix} a & 4 & -1 \\ 3 & a & 2 \\ a & 1 & 1 \end{pmatrix}$. [3]
- 2 The complex number $7 + 3i$ is denoted by z . Find
- (i) $|z|$ and $\arg z$, [2]
- (ii) $\frac{z}{4-i}$, showing clearly how you obtain your answer. [3]
- 3 The matrices \mathbf{A} and \mathbf{B} are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -4 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix}$ and \mathbf{I} is the 2×2 identity matrix. Find
- (i) $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$, [2]
- (ii) \mathbf{A}^{-1} , [2]
- (iii) $(\mathbf{AB}^{-1})^{-1}$. [3]
- 4 (a) Find the matrix that represents a shear with the y -axis invariant, the image of the point $(1, 0)$ being the point $(1, 4)$. [2]
- (b) The matrix \mathbf{X} is given by $\mathbf{X} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$.
- (i) Describe fully the geometrical transformation represented by \mathbf{X} . [2]
- (ii) Find the value of the determinant of \mathbf{X} and describe briefly how this value relates to the transformation represented by \mathbf{X} . [2]
- 5 The cubic equation $2x^3 + 3x + 3 = 0$ has roots α , β and γ .
- (i) Use the substitution $x = u + 2$ to find a cubic equation in u . [3]
- (ii) Hence find the value of $\frac{1}{\alpha-2} + \frac{1}{\beta-2} + \frac{1}{\gamma-2}$. [4]
- 6 (i) Show that $\frac{1}{r^2} - \frac{1}{(r+2)^2} \equiv \frac{4(r+1)}{r^2(r+2)^2}$. [2]
- (ii) Hence find an expression, in terms of n , for $\sum_{r=1}^n \frac{4(r+1)}{r^2(r+2)^2}$. [6]
- (iii) Find $\sum_{r=5}^{\infty} \frac{4(r+1)}{r^2(r+2)^2}$, giving your answer in the form $\frac{p}{q}$ where p and q are integers. [2]

3

- 7 The loci C_1 and C_2 are given by $\arg(z-2-2i) = \frac{1}{4}\pi$ and $|z| = |z-10|$ respectively.
- (i) Sketch on a single Argand diagram the loci C_1 and C_2 . [4]
- (ii) Indicate, by shading, the region of the Argand diagram for which
- $$0 \leq \arg(z-2-2i) \leq \frac{1}{4}\pi \text{ and } |z| \geq |z-10|. \quad [3]$$
- 8 (i) Show that $\sum_{r=n}^{2n} r^3 = \frac{3}{4}n^2(n+1)(5n+1)$. [4]
- (ii) Hence find $\sum_{r=n}^{2n} r(r^2-2)$, giving your answer in a fully factorised form. [5]
- 9 The roots of the equation $x^3 - kx^2 - 2 = 0$ are α , β and γ , where α is real and β and γ are complex.
- (i) Show that $k = \alpha - \frac{2}{\alpha^2}$. [2]
- (ii) Given that $\beta = u + iv$, where u and v are real, find u in terms of α . [4]
- (iii) Find v^2 in terms of α . [4]
- 10 The sequence u_1, u_2, u_3, \dots is defined by $u_n = 5^n + 2^{n-1}$.
- (i) Find u_1, u_2 and u_3 . [2]
- (ii) Hence suggest a positive integer, other than 1, which divides exactly into every term of the sequence. [1]
- (iii) By considering $u_{n+1} + u_n$, prove by induction that your suggestion in part (ii) is correct. [5]

END OF QUESTION PAPER

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