OCR
Oxford Cambridge and RSA

Monday 22 June 2015 – Morning

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 By first expressing $\tanh y$ in terms of exponentials, prove that $\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$. **[3]**

2 It is given that $f(x) = \ln(1+\sin x)$. Using standard series, find the Maclaurin series for $f(x)$ up to and including the term in x^3. **[4]**

3 By first completing the square, find the exact value of $\int_{\frac{1}{2}}^{1} \frac{1}{\sqrt{2x-x^2}}\,dx$. **[5]**

4 It is given that $I_n = \int_0^1 x^n e^{-x} dx$ for $n \geqslant 0$.

(i) Show that $I_n = nI_{n-1} + k$ for $n \geqslant 1$, where k is a constant to be determined. **[3]**

(ii) Find the exact value of I_3. **[3]**

(iii) Find the exact value of $990I_8 - I_{11}$. **[3]**

5 It is given that $y = \sin^{-1}2x$.

(i) Using the derivative of $\sin^{-1}x$ given in the List of Formulae (MF1), find $\frac{dy}{dx}$. **[1]**

(ii) Show that $(1-4x^2)\frac{d^2y}{dx^2} = 4x\frac{dy}{dx}$. **[3]**

(iii) Hence show that $(1-4x^2)\frac{d^3y}{dx^3} - 12x\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = 0$. **[2]**

(iv) Using your results from parts **(i)**, **(ii)** and **(iii)**, find the Maclaurin series for $\sin^{-1}2x$ up to and including the term in x^3. **[3]**

6 It is given that the equation $3x^3 + 5x^2 - x - 1 = 0$ has three roots, one of which is positive.

(i) Show that the Newton-Raphson iterative formula for finding this root can be written

$$x_{n+1} = \frac{6x_n^{\,3} + 5x_n^{\,2} + 1}{9x_n^{\,2} + 10x_n - 1}. \quad \textbf{[3]}$$

(ii) A sequence of iterates x_1, x_2, x_3, ... which will find the positive root is such that the magnitude of the error in x_2 is greater than the magnitude of the error in x_1. On the graph given in the Printed Answer Book, mark a possible position for x_1. **[1]**

(iii) Apply the iterative formula in part **(i)** when the initial value is $x_1 = -1$. Describe the behaviour of the iterative sequence, illustrating your answer on the graph given in the Printed Answer Book. **[2]**

(iv) A sequence of approximations to the positive root is given by x_1, x_2, x_3, ... Successive differences $x_r - x_{r-1} = d_r$, where $r \geqslant 2$, are such that $d_r \approx k(d_{r-1})^2$ where k is a constant.

Show that $d_4 \approx \dfrac{d_3^{\,3}}{d_2^{\,2}}$ and demonstrate this numerically when $x_1 = 1$. **[4]**

(v) Find the value of the positive root correct to 5 decimal places. **[2]**

7 It is given that $\mathrm{f}(x) = \dfrac{x^2 - 25}{(x-1)(x+2)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. **[4]**

(ii) Write down the equations of the asymptotes of the curve $y = \mathrm{f}(x)$. **[2]**

(iii) Find the value of x where the graph of $y = \mathrm{f}(x)$ cuts the horizontal asymptote. **[2]**

(iv) Sketch the graph of $y^2 = \mathrm{f}(x)$. **[2]**

8 It is given that $\mathrm{f}(x) = 2\sinh x + 3\cosh x$.

(i) Show that the curve $y = \mathrm{f}(x)$ has a stationary point at $x = -\frac{1}{2}\ln 5$ and find the value of y at this point. **[4]**

(ii) Solve the equation $\mathrm{f}(x) = 5$, giving your answers exactly. **[5]**

Question 9 begins on page 4.

9 The equation of a curve in polar coordinates is $r = 2\sin 3\theta$ for $0 \leqslant \theta \leqslant \frac{1}{3}\pi$.

(i) Sketch the curve. **[2]**

(ii) Find the area of the region enclosed by this curve. **[4]**

(iii) By expressing $\sin 3\theta$ in terms of $\sin\theta$, show that a cartesian equation for the curve is

$$(x^2 + y^2)^2 = 6x^2y - 2y^3.$$ **[5]**

END OF QUESTION PAPER