OCR
RECOGNISING ACHIEVEMENT

Friday 18 January 2013 – Afternoon

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Find $\int x\cos 3x\,\mathrm{d}x$. **[4]**

2 Find the first three terms in the expansion of $(9-16x)^{\frac{3}{2}}$ in ascending powers of x, and state the set of values for which this expansion is valid. **[5]**

3 The equation of a curve is $xy^2 = x^2 + 1$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y, and hence find the coordinates of the stationary points on the curve. **[7]**

4 The equations of two lines are

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \text{ and } \mathbf{r} = 6\mathbf{i} + 8\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}).$$

(i) Show that these lines meet, and find the coordinates of the point of intersection. **[5]**

(ii) Find the acute angle between these lines. **[3]**

5 The parametric equations of a curve are

$$x = 2 + 3\sin\theta \text{ and } y = 1 - 2\cos\theta \text{ for } 0 \leqslant \theta \leqslant \tfrac{1}{2}\pi.$$

(i) Find the coordinates of the point on the curve where the gradient is $\frac{1}{2}$. **[5]**

(ii) Find the cartesian equation of the curve. **[2]**

6 Use the substitution $u = 2x + 1$ to evaluate $\displaystyle\int_0^{\frac{1}{2}} \frac{4x-1}{(2x+1)^5}\,\mathrm{d}x$. **[7]**

7 **(i)** Given that $y = \ln(1+\sin x) - \ln(\cos x)$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\cos x}$. **[4]**

(ii) Using this result, evaluate $\displaystyle\int_0^{\frac{1}{3}\pi} \sec x\,\mathrm{d}x$, giving your answer as a single logarithm. **[3]**

8 The points A (3, 2, 1), B (5, 4, −3), C (3, 17, −4) and D (1, 6, 3) form a quadrilateral $ABCD$.

(i) Show that $AB = AD$. **[2]**

(ii) Find a vector equation of the line through A and the mid-point of BD. **[3]**

(iii) Show that C lies on the line found in part **(ii)**. **[1]**

(iv) What type of quadrilateral is $ABCD$? **[1]**

9 The temperature of a freezer is –20 °C. A container of a liquid is placed in the freezer. The rate at which the temperature, θ °C, of a liquid decreases is proportional to the difference in temperature between the liquid and its surroundings. The situation is modelled by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta + 20),$$

where time t is in minutes and k is a positive constant.

(i) Express θ in terms of t, k and an arbitrary constant. **[3]**

Initially the temperature of the liquid in the container is 40 °C and, at this instant, the liquid is cooling at a rate of 3 °C per minute. The liquid freezes at 0 °C.

(ii) Find the value of k and find also the time it takes (to the nearest minute) for the liquid to freeze. **[5]**

The procedure is repeated on another occasion with a different liquid. The initial temperature of this liquid is 90 °C. After 19 minutes its temperature is 0 °C.

(iii) Without any further calculation, explain what you can deduce about the value of k in this case. **[1]**

10 **(i)** Use algebraic division to express $\dfrac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}$ in the form $Ax + B + \dfrac{Cx + D}{x^2 - x - 6}$, where A, B, C and D are constants. **[4]**

(ii) Hence find $\displaystyle\int_4^6 \frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}\,\mathrm{d}x$, giving your answer in the form $a + \ln b$. **[7]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.