OCR
RECOGNISING ACHIEVEMENT

Monday 10 June 2013 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The complex number $3 + ai$, where a is real, is denoted by z. Given that $\arg z = \frac{1}{6}\pi$, find the value of a and hence find $|z|$ and $z^* - 3$. **[6]**

2 The matrices **A**, **B** and **C** are given by $\mathbf{A} = (5 \quad 1)$, $\mathbf{B} = (2 \quad -5)$ and $\mathbf{C} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

(i) Find $3\mathbf{A} - 4\mathbf{B}$. **[2]**

(ii) Find **CB**. Determine whether **CB** is singular or non-singular, giving a reason for your answer. **[5]**

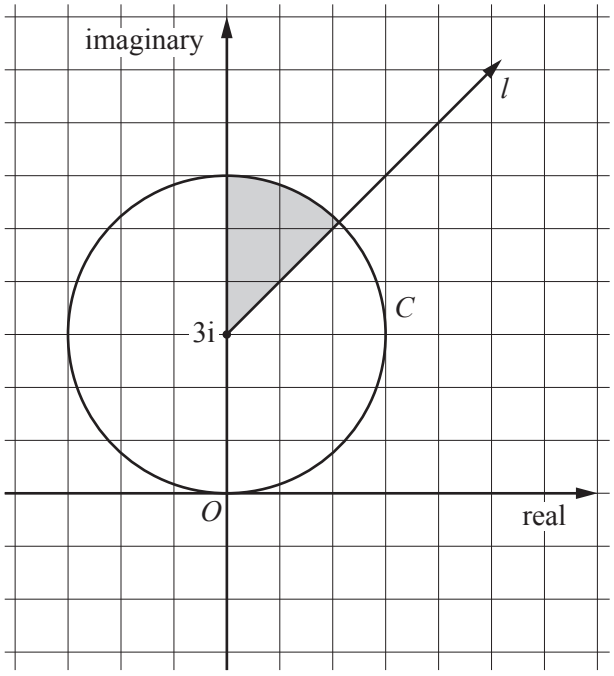
3 Use an algebraic method to find the square roots of $11 + (12\sqrt{5})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[6]**

4 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \geqslant 1$,

$$\mathbf{M}^n = \begin{pmatrix} 2^n & 2^{n+1} - 2 \\ 0 & 1 \end{pmatrix}.$$ **[6]**

5 Find $\sum_{r=1}^{n} (4r^3 - 3r^2 + r)$, giving your answer in a fully factorised form. **[6]**

6



The Argand diagram above shows a half-line l and a circle C. The circle has centre 3i and passes through the origin.

(i) Write down, in complex number form, the equations of l and C. **[4]**

(ii) Write down inequalities that define the region shaded in the diagram. [The shaded region includes the boundaries.] **[3]**

7 **(i)** Find the matrix that represents a rotation through 90° clockwise about the origin. **[2]**

(ii) Find the matrix that represents a reflection in the x-axis. **[2]**

(iii) Hence find the matrix that represents a rotation through 90° clockwise about the origin, followed by a reflection in the x-axis. **[2]**

(iv) Describe a **single** transformation that is represented by your answer to part **(iii)**. **[2]**

8 The cubic equation $kx^3 + 6x^2 + x - 3 = 0$, where k is a non-zero constant, has roots α, β and γ.

Find the value of $(\alpha + 1)(\beta + 1) + (\beta + 1)(\gamma + 1) + (\gamma + 1)(\alpha + 1)$ in terms of k. **[6]**

9 **(i)** Show that $\dfrac{1}{3r-1} - \dfrac{1}{3r+2} \equiv \dfrac{3}{(3r-1)(3r+2)}$. **[2]**

(ii) Hence show that $\displaystyle\sum_{r=1}^{2n} \frac{1}{(3r-1)(3r+2)} = \frac{n}{2(3n+1)}$. **[6]**

10 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 2 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 1 \end{pmatrix}$.

(i) Find the value of a for which $\mathbf{A}$ is singular. **[5]**

(ii) Given that $\mathbf{A}$ is non-singular, find $\mathbf{A}^{-1}$ and hence solve the equations

$$\begin{aligned} ax + 2y + z &= 1, \\ x + 3y + 2z &= 2, \\ 4x + y + z &= 3. \end{aligned}$$

[7]

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE