



Monday 23 June 2014 – Morning

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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1 Find $\int_0^2 \frac{1}{\sqrt{4+x^2}} dx$, giving your answer exactly in logarithmic form. [3]

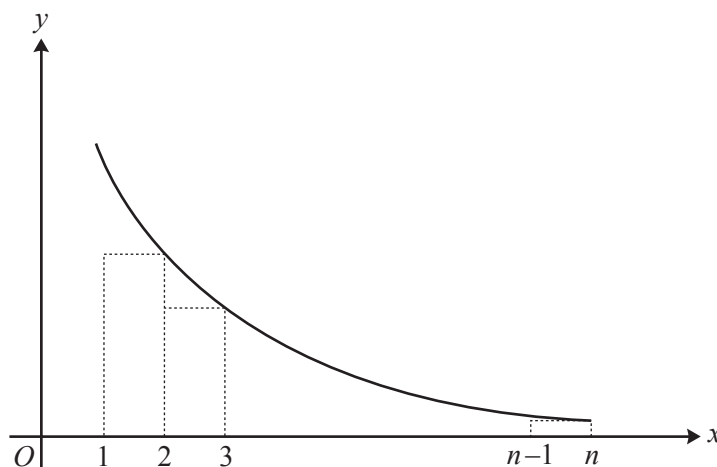
2 It is given that $f(x) = \ln(1+x^2)$.

(i) Using the standard Maclaurin expansion for $\ln(1+x)$, write down the first four terms in the expansion of $f(x)$, stating the set of values of x for which the expansion is valid. [3]

(ii) Hence find the exact value of

$$1 - \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{3}\left(\frac{1}{2}\right)^4 - \frac{1}{4}\left(\frac{1}{2}\right)^6 + \dots$$
 [2]

3 The diagram shows the curve $y = \frac{1}{x^3}$ for $1 \leq x \leq n$ where n is an integer. A set of $(n-1)$ rectangles of unit width is drawn under the curve.



(i) Write down the sum of the areas of the rectangles. [2]

(ii) Hence show that $\sum_{r=1}^{\infty} \frac{1}{r^3} < \frac{3}{2}$. [5]

4 The curves $y = \cos^{-1}x$ and $y = \tan^{-1}(\sqrt{2}x)$ intersect at a point A .

(i) Verify that the coordinates of A are $(\frac{1}{\sqrt{2}}, \frac{1}{4}\pi)$. [2]

(ii) Determine whether the tangents to the curves at A are perpendicular. [4]

3

- 5 A curve has equation $y = \frac{x^2 - 8}{x - 3}$.
- (i) Find the equations of the asymptotes of the curve. [3]
- (ii) Prove that there are no points on the curve for which $4 < y < 8$. [4]
- (iii) Sketch the curve. Indicate the asymptotes in your sketch. [2]
- 6 (i) Given that $y = \cosh^{-1}x$, show that $y = \ln(x + \sqrt{x^2 - 1})$. [4]
- (ii) Show that $\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2 - 1}}$. [2]
- (iii) Solve the equation $\cosh x = 3$, giving your answers in logarithmic form. [3]
- 7 It is given that, for non-negative integers n , $I_n = \int_0^{\frac{1}{2}\pi} \sin^n x \, dx$.
- (i) Show that $I_n = \frac{n-1}{n} I_{n-2}$ for $n \geq 2$. [3]
- (ii) Explain why $I_{2n+1} < I_{2n-1}$. [2]
- (iii) It is given that $I_{2n+1} < I_{2n} < I_{2n-1}$. Take $n = 5$ to find an interval within which the value of π lies. [6]
- 8 A curve has polar equation $r = a(1 + \cos \theta)$, where a is a positive constant and $0 \leq \theta < 2\pi$.
- (i) Find the equation of the tangent at the pole. [2]
- (ii) Sketch the curve. [2]
- (iii) Find the area enclosed by the curve. [6]
- 9 The equation $10x - 8 \ln x = 28$ has a root α in the interval $[3, 4]$. The iteration $x_{n+1} = g(x_n)$, where $g(x) = 2.8 + 0.8 \ln x$ and $x_1 = 3.8$, is to be used to find α .
- (i) Find the value of α correct to 5 decimal places. You should show the result of each step of the iteration to 6 decimal places. [4]
- (ii) Illustrate this iteration by means of a sketch. [2]
- (iii) The difference, δ_r , between successive approximations is given by $\delta_r = x_{r+1} - x_r$. Find δ_3 . [2]
- (iv) Given that $\delta_{n+1} \approx g'(\alpha)\delta_n$, for all positive integers n , estimate the smallest value of n such that $\delta_n < 10^{-6}\delta_1$. [4]

END OF QUESTION PAPER

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