OCR

Monday 23 June 2014 – Morning

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Find $\int_0^2 \frac{1}{\sqrt{4+x^2}}\,\mathrm{d}x$, giving your answer exactly in logarithmic form. **[3]**

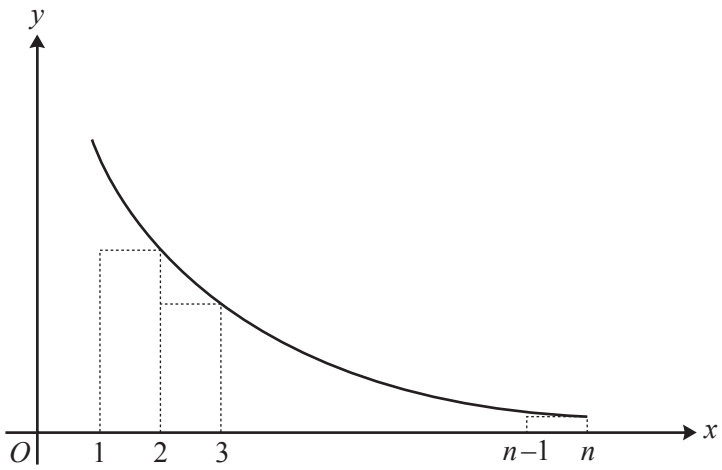
2 It is given that $\mathrm{f}(x) = \ln(1+x^2)$.

(i) Using the standard Maclaurin expansion for $\ln(1+x)$, write down the first four terms in the expansion of $\mathrm{f}(x)$, stating the set of values of x for which the expansion is valid. **[3]**

(ii) Hence find the exact value of

$$1-\frac{1}{2}\left(\frac{1}{2}\right)^2+\frac{1}{3}\left(\frac{1}{2}\right)^4-\frac{1}{4}\left(\frac{1}{2}\right)^6+\ldots. \quad \textbf{[2]}$$

3 The diagram shows the curve $y=\frac{1}{x^3}$ for $1 \leqslant x \leqslant n$ where n is an integer. A set of $(n-1)$ rectangles of unit width is drawn under the curve.



(i) Write down the sum of the areas of the rectangles. **[2]**

(ii) Hence show that $\sum_{r=1}^{\infty}\frac{1}{r^3} < \frac{3}{2}$. **[5]**

4 The curves $y=\cos^{-1}x$ and $y=\tan^{-1}(\sqrt{2}x)$ intersect at a point A.

(i) Verify that the coordinates of A are $\left(\frac{1}{\sqrt{2}}, \frac{1}{4}\pi\right)$. **[2]**

(ii) Determine whether the tangents to the curves at A are perpendicular. **[4]**

5 A curve has equation $y = \dfrac{x^2 - 8}{x - 3}$.

(i) Find the equations of the asymptotes of the curve. **[3]**

(ii) Prove that there are no points on the curve for which $4 < y < 8$. **[4]**

(iii) Sketch the curve. Indicate the asymptotes in your sketch. **[2]**

6 **(i)** Given that $y = \cosh^{-1}x$, show that $y = \ln\left(x + \sqrt{x^2 - 1}\right)$. **[4]**

(ii) Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\cosh^{-1}x\right) = \dfrac{1}{\sqrt{x^2 - 1}}$. **[2]**

(iii) Solve the equation $\cosh x = 3$, giving your answers in logarithmic form. **[3]**

7 It is given that, for non-negative integers n, $I_n = \displaystyle\int_0^{\frac{1}{2}\pi} \sin^n x \,\mathrm{d}x$.

(i) Show that $I_n = \dfrac{n-1}{n} I_{n-2}$ for $n \geqslant 2$. **[3]**

(ii) Explain why $I_{2n+1} < I_{2n-1}$. **[2]**

(iii) It is given that $I_{2n+1} < I_{2n} < I_{2n-1}$. Take $n = 5$ to find an interval within which the value of π lies. **[6]**

8 A curve has polar equation $r = a(1 + \cos\theta)$, where a is a positive constant and $0 \leqslant \theta < 2\pi$.

(i) Find the equation of the tangent at the pole. **[2]**

(ii) Sketch the curve. **[2]**

(iii) Find the area enclosed by the curve. **[6]**

9 The equation $10x - 8\ln x = 28$ has a root α in the interval [3, 4]. The iteration $x_{n+1} = \mathrm{g}(x_n)$, where $\mathrm{g}(x) = 2.8 + 0.8\ln x$ and $x_1 = 3.8$, is to be used to find α.

(i) Find the value of α correct to 5 decimal places. You should show the result of each step of the iteration to 6 decimal places. **[4]**

(ii) Illustrate this iteration by means of a sketch. **[2]**

(iii) The difference, δ_r, between successive approximations is given by $\delta_r = x_{r+1} - x_r$. Find δ_3. **[2]**

(iv) Given that $\delta_{n+1} \approx \mathrm{g}'(\alpha)\delta_n$, for all positive integers n, estimate the smallest value of n such that $\delta_n < 10^{-6}\delta_1$. **[4]**

END OF QUESTION PAPER

