RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4723:** Core Mathematics 3

Mark Scheme for January 2012

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation in scoris	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			State $2\ln x$ Use both relevant logarithm properties correctly Obtain $\ln 3$	B1 M1 A1 **[3]**	may be implied by immediate use of limits either or both may be implied, eg by $2\ln\sqrt{6} = \ln 6$ or by $\ln 6 - \ln 2 = \ln 3$ AG; with at least one property shown explicitly	
2			State volume is $\int \frac{36\pi}{(2x+1)^4}\,\mathrm{d}x$ Obtain integral of form $k(2x+1)^n$ Obtain $-6\pi(2x+1)^{-3}$ or $-6(2x+1)^{-3}$ Substitute correct limits and subtract Obtain $\frac{52}{9}\pi$	B1 M1 A1 M1 A1 **[5]**	or equiv in terms of x; no need for limits; condone absence of dx; condone absence of π here if it appears later in solution (even as part of a wrong answer) for any $n \le -1$; with or without π; or ku^n following substitution; allow if $n = -5$; allow M1 if one slight slip occurs in $(2x+1)$ or (unsimplified) equiv the correct way round for integral of form $k(2x+1)^{-3}$; allow if one slight slip occurs in $(2x+1)$; not earned if limit 0 leads to $\ldots - 0$ or similarly simplified exact equiv	

Question			Answer	Marks	Guidance	
3			Attempt use of quotient rule	M1	condone *u*/*v* muddles but needs $(x+2)^2$ in denominator; condone numerator back to front; or product rule to produce terms involving $(x+2)^{-1}$ and $(x+2)^{-2}$	
			Obtain $\frac{2x(x+2)-(x^2+4)}{(x+2)^2}$	A1	or equiv; brackets may be implied by subsequent recovery	
			Substitute 1 into attempt at first derivative	M1	also allow if sign slip leads to derivative cancelling to 1	
			Obtain $\frac{1}{9}$	A1		
			Use –9 as gradient of normal	A1ft	following their value of first derivative	
			Attempt to find equation of normal	M1	not equation of tangent; needs use of negative reciprocal of their derivative value	
			Obtain $27x+3y-32=0$	A1	or equiv of requested form	
				[7]		
4	(i)		State $\tan\alpha = 2$	B1	ignoring subsequent work to find angle	
			Use identity $\sec^2\beta = 1+\tan^2\beta$	B1		
			Attempt solution of quad eqn for $\tan\beta$	M1	3 term quad eqn; using reasonable attempt at factorisation to find value or use of quadratic formula (with no more than one slip)	
			Obtain $\tan\beta = 5$	A1	ignoring subsequent work to find angle; value 5 must be obtained legitimately	
				[4]		

Question			Answer	Marks	Guidance	
4	(ii)		Substitute their values of $\tan\alpha$ and $\tan\beta$ in formula … Obtain $\frac{2+5}{1-2\times5}$ Obtain $-\frac{7}{9}$	M1 A1ft A1 **[3]**	… of form $\frac{\pm\tan\alpha\pm\tan\beta}{\pm1\pm\tan\alpha\tan\beta}$ following their values from part **(i)** or correct simplified exact equiv including $\frac{7}{-9}$; A0 if $\tan\beta=5$ obtained incorrectly in part **(i)** SC: use of calculator for $\tan(\tan^{-1}2+\tan^{-1}5)$ to give $-\frac{7}{9}$ earns all 3 marks (but 0 out of 3 if answer is not exact); with either or both of 2 and 5 wrong, 2 out of 3 available for this approach if result is exact and correct given their two values	
5	(i)		State 26 State 4	B1 B1 **[2]**		
5	(ii)		Sketch (more or less) correct curve Refer to reflection in $y=x$ or symmetrical about $y=x$ or mirrored in $y=x$	B1 B1 **[2]**	with approx correct curvatures and curve going through second quadrant but not fourth quadrant; allow if sketch does not meet given curve on line $y=x$ explicit reference needed, not just line $y=x$ shown on sketch	

Question			Answer	Marks	Guidance	
5	(iii)		Attempt calculation $k(y+4y+2y+...)$	M1	any constant *k*; with *y*-values from table and coefficients 1, 2 and 4 occurring at least once each; brackets may be implied by subsequent calculation	
			Obtain $k(1+32+28+76+46+100+26)$	A1	or (unsimplified) equiv	
			Use $k=\frac{1}{3}\times 2$	A1		
			Obtain 206	A1 **[4]**		
6	(i)		Obtain rational expression of form $\frac{\mathrm{f}(y)}{y^3+2y}$	M1	where $\mathrm{f}(y)$ is not constant; ignore how expression is labelled	
			Obtain $\frac{3y^2+2}{y^3+2y}$	A1 **[2]**		
6	(ii)		Recognise that $\frac{\mathrm{d}y}{\mathrm{d}x}=1\div\frac{\mathrm{d}x}{\mathrm{d}y}$ for rational expression of form $\frac{\mathrm{f}(y)}{y^3+2y}$	M1	may be implied	
			Obtain $\frac{y^3+2y}{3y^2+2}=4$ or $\frac{3y^2+2}{y^3+2y}=\frac{1}{4}$	A1ft	following their rational expression from **(i)**	
			Confirm $y=\frac{12y^2+8}{y^2+2}$	A1 **[3]**	AG; following correct work and with at least one step between $\frac{y^3+2y}{3y^2+2}=4$ or equiv and answer	

Question			Answer	Marks	Guidance	
6	(iii)		Obtain correct first iterate 11.89	B1	or greater accuracy; having started with 12; accept if 12 used in part **(ii)** to produce next value and 11.89 used as starting value here	
			Attempt iteration process to produce at least 3 iterates in all	M1	implied by plausible sequence of values; having started anywhere; if formula clearly not based on equation from part **(ii)**, award M0	
			Obtain at least 2 more correct iterates	A1	showing at least 3 decimal places	
			Obtain 11.888 for y	A1	answer needed to exactly 3 decimal places	
			Obtain 7.441 for x	A1	answer needed to exactly 3 decimal places; award final A0 if not clear which is x and which is y	
					[12 → 11.89041 → 11.88841 → 11.88837]	
				[5]		

Question			Answer	Marks	Guidance	
7	(i)	(a)	State or imply $e^{-0.132t} = 0.25$ Attempt solution of eqn of form $e^{-0.132t} = k$ Obtain 10.5	B1 M1 A1 **[3]**	or equiv such as $40e^{-0.132t} = 10$ using sound process; implied by correct ans; allow trial and improvement attempt or greater accuracy	
7	(i)	(b)	Differentiate to obtain $ke^{-0.132t}$ Obtain $5.28e^{-0.132t}$ or $-5.28e^{-0.132t}$ Substitute 5 to obtain 2.73 or –2.73	M1 A1 A1 **[3]**	where k is a constant not equal to 40 (allow even if process looks like integration) or (unsimplified) equiv accept 2.7 or –2.7 or greater accuracy; allow 2.73 or –2.73 whatever it is claimed to be	
7	(ii)		EITHER Attempt to solve $40e^{2\lambda} = 31.4$ or $40e^{-2\lambda} = 31.4$ Obtain or imply $40e^{-0.121t}$ Substitute 3 to obtain 27.8 OR Attempt calculation involving multiplication of power of $\frac{31.4}{40}$ Obtain $31.4\times(\frac{31.4}{40})^{0.5}$ or $40\times(\frac{31.4}{40})^{1.5}$ Obtain 27.8	 M1 A1 A1 **[3]** M1 A1 A1	 using sound process; method implied by correct formula for mass of B obtained or greater accuracy (–0.12103..) or $0.5\ln 0.785$ accept 28 or greater accuracy accept 28 or greater accuracy	

Question			Answer	Marks	Guidance	
8	(i)		State $\cos 4\theta = 1-2\sin^2 2\theta$ State or clearly imply $\sin 2\theta = 2\sin\theta\cos\theta$ Obtain $1-8\sin^2\theta\cos^2\theta$	B1 B1 B1 **[3]**	possibly substituted in incorrect expression	
8	(ii)		Produce expression involving $\cos\frac{4}{24}\pi$ as only trigonometrical ratio Obtain $\frac{1}{8}-\frac{1}{16}\sqrt{3}$	M1 A1 **[2]**	or exact equiv (including, eg $\frac{1-\frac{1}{2}\sqrt{3}}{8}$)	
8	(iii)		Use $2\cos^2 2\theta = 1+\cos 4\theta$ Attempt to express in terms of $\cos 4\theta$ Obtain $\frac{2}{3}+\frac{4}{3}\cos 4\theta$ Substitute at least one of -1 and 1 for $\cos 4\theta$ in expression where $\cos 4\theta$ is only trigonometrical ratio Obtain 2 and $-\frac{2}{3}$	B1 M1 A1 M1 A1 **[5]**	or use $2\cos^2 2\theta = 2-8\sin^2\theta\cos^2\theta$ or unsimplified equiv or at least one of $\theta = \frac{1}{4}\pi$ and $\theta = 0$	

Question			Answer	Marks	Guidance	
9	(i)		Attempt differentiation to find *x*-coordinate of stationary point or attempt completion of square as far as $(x+...)^2$	M1	or equiv; first two marks of part **(i)** may be earned by work seen in part **(ii)**; $x=-2$ only stated earns M1A1	
			Obtain $x=-2$ or $(x+2)^2$	A1	first two marks of part **(i)** are implied by correct answer to translation in *x*-direction	
			State translation by 2 in negative *x*-direction	A1	or (clear) equiv; allow correct vector	
			State translation by 4 in negative *y*-direction	A1	or (clear) equiv; allow correct vector	
			State stretch parallel to *y*-axis, scale factor *k*	B1	or equiv at least mentioning *y* and *k*	
				[5]		
9	(ii)		State one of $y<4k$, $y\le 4k$, $y<-4k$, $y\le -4k$ $y>4k$, $y\ge 4k$, $y>-4k$, $y\ge -4k$	B1		
			State $y\ge -4k$	B1	allow alternative notation such as $f(x)\ge -4k$ or range $\ge -4k$	
				[2]		
9	(iii)		Attempt to relate *y*-value involving *k* at their stationary point to 20 or -20 or consider discriminant of $k(x^2+4x)=20$ or of $k(x^2+4x)=-20$	*M1		
			Obtain $k=5$	A1	earned unless there is clear evidence of error in working	
			State one root $x=-2$	B1		
			Attempt solution of $k(x^2+4x)=20$	M1	dep *M; for their value of *k* provided positive	
			Obtain $\frac{-4\pm\sqrt{32}}{2}$	A1ft	or (unsimplified) exact equivs; following their value of *k*	
			Obtain $-2\pm 2\sqrt{2}$ or $-2\pm\sqrt{8}$	A1	dependent on previous A1 A1ft marks being awarded	
				[6]		

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